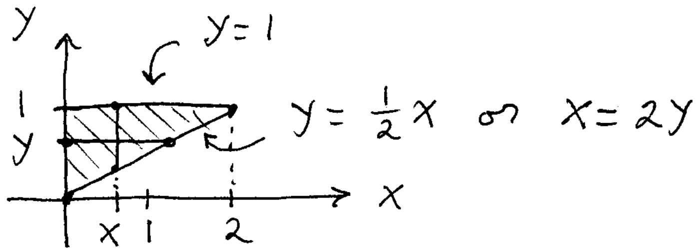
Section 15.2

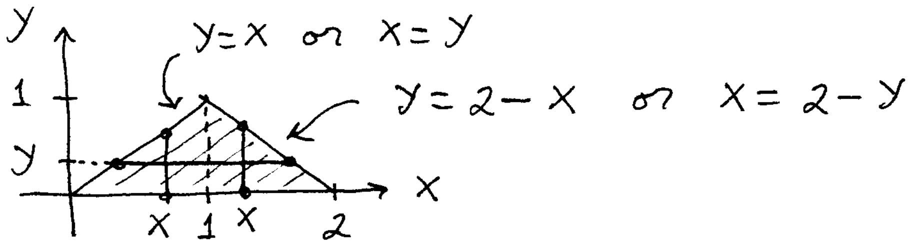
1.)



a.) $0 \le x \le 2$ and $\frac{1}{2}x \le y \le 1$

b.) $0 \le y \le 1$ and $0 \le x \le 2y$

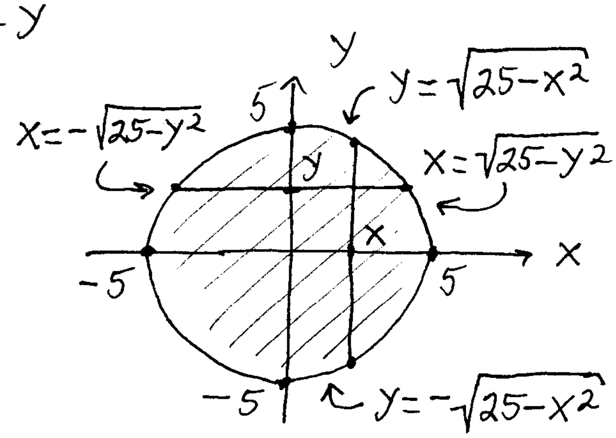
2.)



a.) $0 \le x \le 1$ and $0 \le y \le x$; $1 \le x \le 2$ and $0 \le y \le 2-x$

b.) $0 \le y \le 1$ and $y \le x \le 2-y$

5.) circle $x^2+y^2=25$;



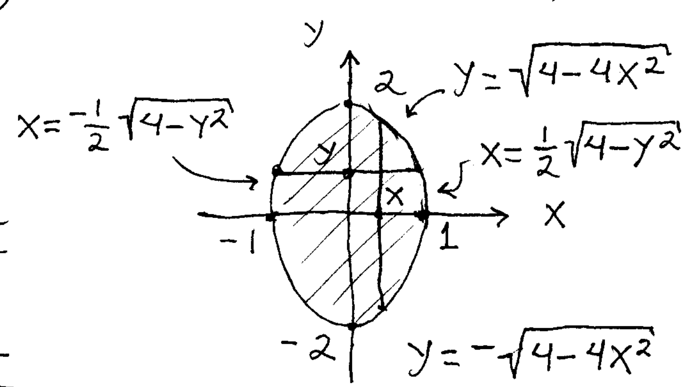
a.) $-5 \le x \le 5$ and

$-\sqrt{25-x^2} \le y \le +\sqrt{25-x^2}$

b.) $-5 \le y \le 5$ and

$-\sqrt{25-y^2} \le x \le +\sqrt{25-y^2}$

8.) ellipse $4x^2+y^2=4$;



a.) $-1 \le x \le 1$ and

$-\sqrt{4-4x^2} \le y \le \sqrt{4-4x^2}$

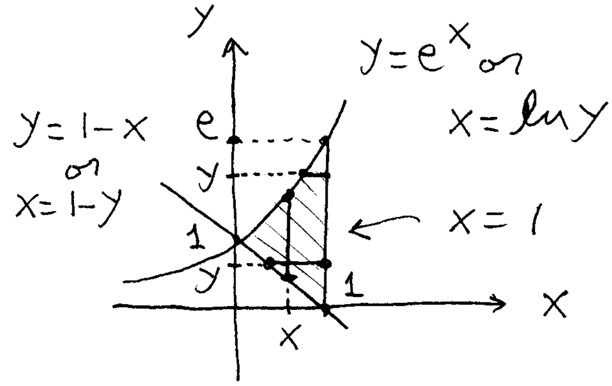
b.) $-2 \le y \le 2$ and

$-\frac{1}{2}\sqrt{4-y^2} \le x \le \frac{1}{2}\sqrt{4-y^2}$

10.) a.) $0 \le x \le 1$ and $1 - x \le y \le e^x$

b.) $0 \le y \le 1$ and $1 - y \le x \le 1$;

$1 \le y \le e$ and $\ln y \le x \le 1$

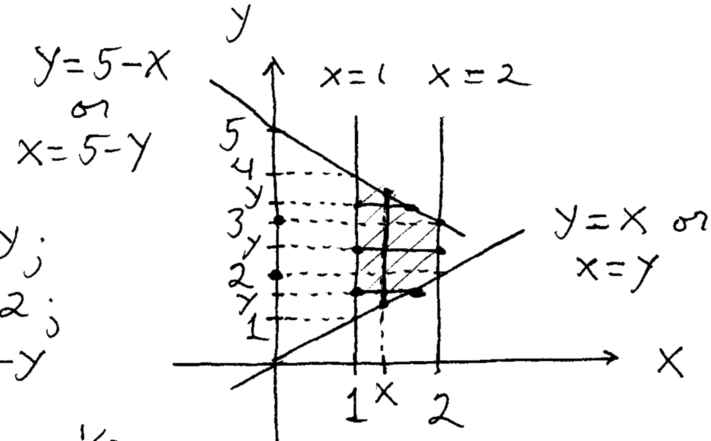


12.) a.) $1 \le x \le 2$ and $x \le y \le 5 - x$

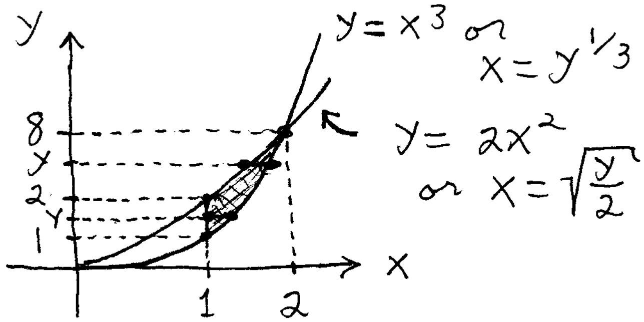
b.) $1 \le y \le 2$ and $1 \le x \le y$;

$2 \le y \le 3$ and $1 \le x \le 2$;

$3 \le y \le 4$ and $1 \le x \le 5 - y$



14.)



$1 \le y \le 2$ and $1 \le x \le y^{1/3}$;

$2 \le y \le 8$ and $\sqrt{y/2} \le x \le y^{1/3}$

17.) $\int_0^1 \int_0^x (x + 2y)\, dy\, dx = \int_0^1 (xy + y^2)\Big|_{y=0}^{y=x} dx$

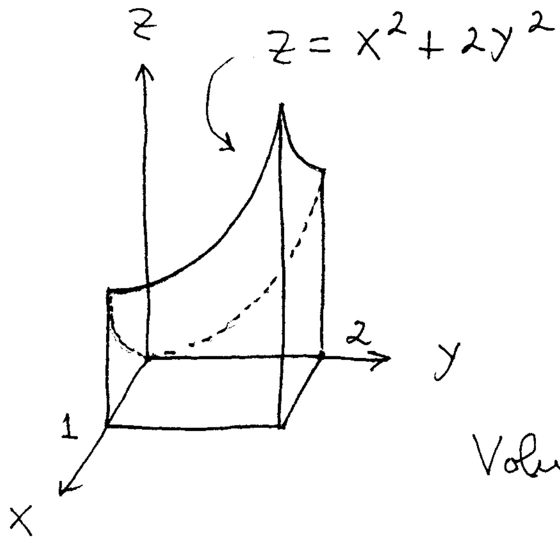
$= \int_0^1 (2x^2 - 0)\, dx = \frac{2}{3}x^3 \Big|_0^1 = \frac{2}{3}(1) - \frac{2}{3}(0) = \frac{2}{3}$

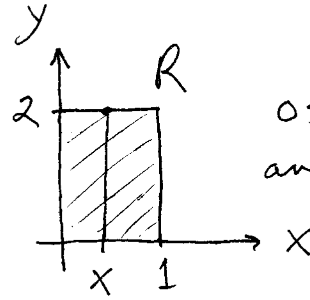
20.) $\int_1^2 \int_0^y e^{x+y}\, dx\, dy = \int_1^2 e^{x+y}\Big|_{x=0}^{x=y} dy$

$= \int_1^2 (e^{2y} - e^y)\, dy = (\frac{1}{2}e^{2y} - e^y)\Big|_1^2$

$= (\frac{1}{2}e^4 - e^2) - (\frac{1}{2}e^2 - e) = \frac{1}{2}e^4 - \frac{3}{2}e^2 + e$

24.)

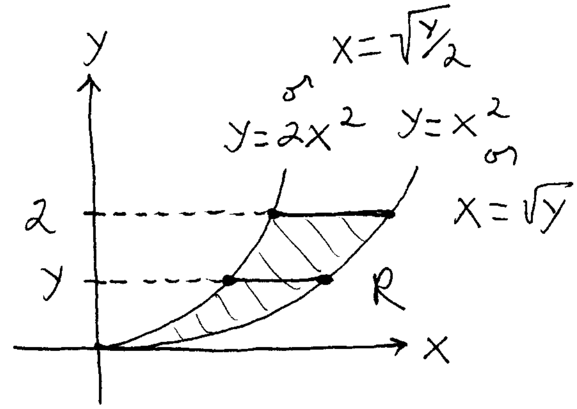


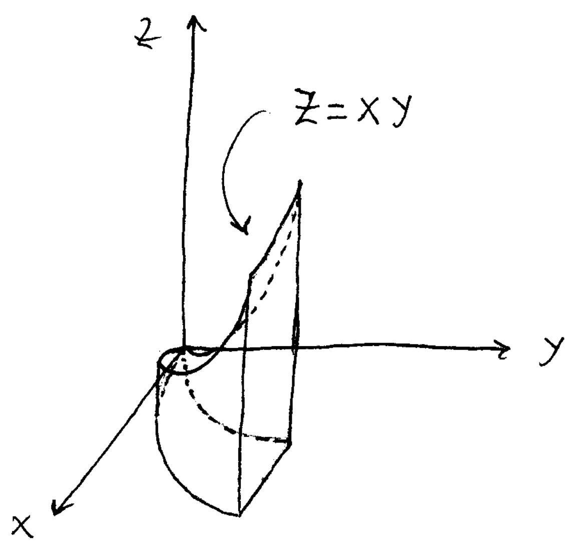


$0 \le x \le 1$ and $0 \le y \le 2$

$$\text{Volume} = \int_0^1 \int_0^2 (x^2+2y^2)\, dy\, dx$$

26.)

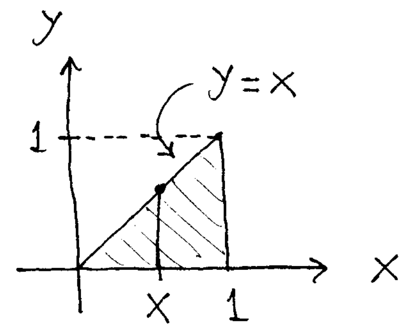




$0 \le y \le 2$ and $\sqrt{\frac{y}{2}} \le x \le \sqrt{y}$

$$\text{Volume} = \int_0^2 \int_{\sqrt{y/2}}^{\sqrt{y}} xy\, dx\, dy$$

28.) density $f(x,y) = \frac{1}{1+x^2} \rightarrow$

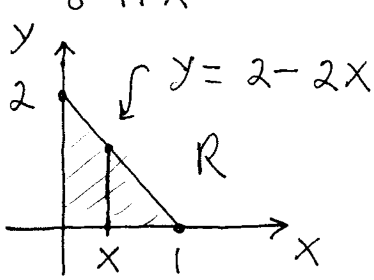


$$\text{Mass} = \int_0^1 \int_0^x \frac{1}{1+x^2}\, dy\, dx$$

$$= \int_0^1 \left(\frac{y}{1+x^2} \Big|_{y=0}^{y=x} \right) dx$$

$$= \int_0^1 \frac{x}{1+x^2}\, dx = \frac{1}{2}\ln(1+x^2)\Big|_0^1 = \frac{1}{2}\ln 2 - \frac{1}{2}\underbrace{\ln 1}_{0} = \frac{1}{2}\ln 2$$

29.)



temperature
$T(x,y) = \cos(x+2y)$,
area of $R = 1$, then

$$AVE = \frac{1}{\text{area}R} \int_0^1 \int_0^{2-2x} \cos(x+2y)\,dy\,dx$$

$$= \frac{1}{1} \int_0^1 \left(\frac{1}{2} \sin(x+2y) \Big|_{y=0}^{y=2-2x} \right) dx$$

$$= \int_0^1 \left(\frac{1}{2} \sin(x+2(2-2x)) - \frac{1}{2} \sin x \right) dx$$

$$= \int_0^1 \left(\frac{1}{2} \sin(4-3x) - \frac{1}{2} \sin x \right) dx$$

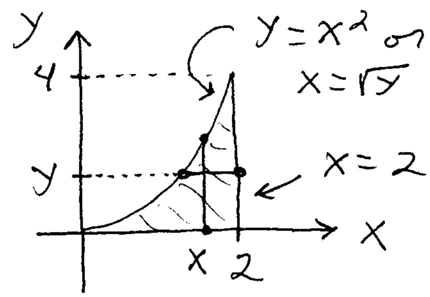
$$= \left[\frac{1}{2} \cdot \frac{1}{3} \cos(4-3x) + \frac{1}{2} \cos x \right] \Big|_0^1$$

$$= \left(\frac{1}{6} \cos 1 + \frac{1}{2} \cos 1 \right) - \left(\frac{1}{6} \cos 4 + \frac{1}{2} \cancel{\cos 0}^{\,1} \right)$$

$$= \frac{2}{3} \cos 1 - \frac{1}{6} \cos 4 - \frac{1}{2}$$

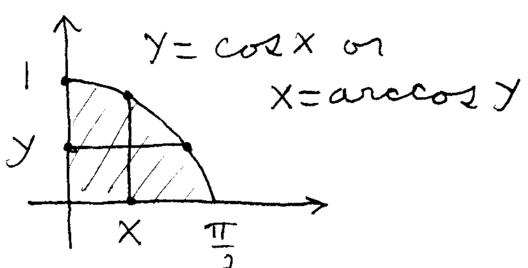
31.) $\int_0^2 \int_0^{x^2} x^3 y\,dy\,dx$ $\qquad$ $0 \le x \le 2$ and $0 \le y \le x^2$;

$= \int_0^4 \int_{\sqrt{y}}^2 x^3 y\,dx\,dy$ $\qquad$ $0 \le y \le 4$ and $\sqrt{y} \le x \le 2$



32.) $0 \le x \le \frac{\pi}{2}$ and $0 \le y \le \cos x$;

$0 \le y \le 1$ and $0 \le x \le \arccos y$,



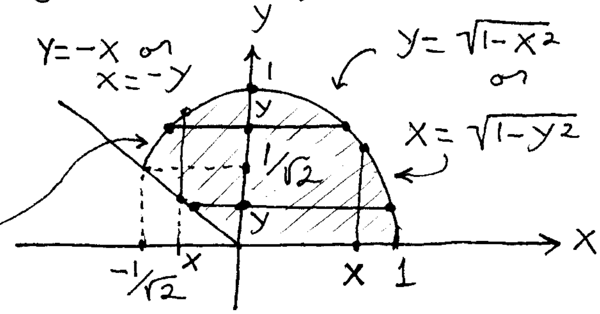
$$\int_0^{\pi/2} \int_0^{\cos x} x^2\,dy\,dx$$

$$= \int_0^1 \int_0^{\arccos y} x^2\,dx\,dy$$

34.) $\int_{-1/\sqrt{2}}^0 \int_{-x}^{\sqrt{1-x^2}} x^3 y\,dy\,dx + \int_0^1 \int_0^{\sqrt{1-x^2}} x^3 y\,dy\,dx$

$-1/\sqrt{2} \le x \le 0$ and $-x \le y \le \sqrt{1-x^2}$,

$0 \le x \le 1$ and $0 \le y \le \sqrt{1-x^2}$;



then $0 \le y \le \frac{1}{\sqrt{2}}$ and $-y \le x \le \sqrt{1-y^2}$;

$\frac{1}{\sqrt{2}} \le y \le 1$ and $-\sqrt{1-y^2} \le x \le +\sqrt{1-y^2}$,

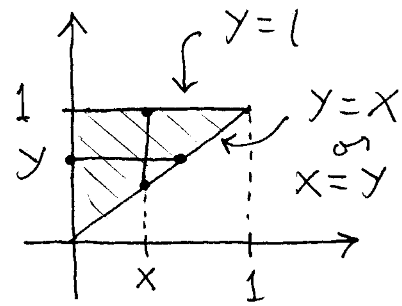
so that

$$\int_{-\frac{1}{\sqrt{2}}}^{0} \int_{-x}^{\sqrt{1-x^2}} x^3 y \, dy \, dx + \int_0^1 \int_0^{\sqrt{1-x^2}} x^3 y \, dy \, dx$$

$$= \int_0^{\frac{1}{\sqrt{2}}} \int_{-y}^{\sqrt{1-y^2}} x^3 y \, dx \, dy + \int_{\frac{1}{\sqrt{2}}}^{1} \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} x^3 y \, dx \, dy .$$

35.) $\int_0^1 \int_x^1 \sin y^2 \, dy \, dx$ $\quad 0 \le x \le 1,\ x \le y \le 1$;

$0 \le y \le 1,$
$0 \le x \le y$



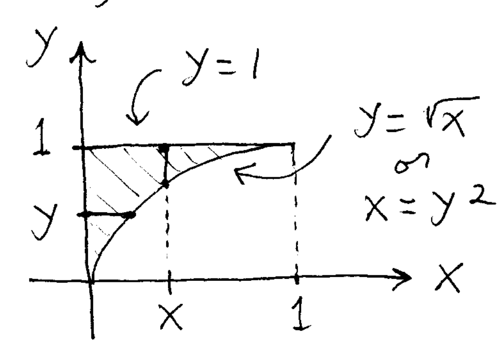
$$= \int_0^1 \int_0^y \sin y^2 \, dx \, dy$$

$$= \int_0^1 \left(x \cdot \sin y^2\right)\Big|_{x=0}^{x=y} dy$$

$$= \int_0^1 y \sin y^2 \, dy$$

$$= -\frac{1}{2} \cos y^2 \Big|_0^1 = \left(\frac{-1}{2}\cos 1\right) - \left(\frac{-1}{2}\cos 0\right) = \frac{1}{2} - \frac{1}{2}\cos 1$$

36.) $\int_0^1 \int_{\sqrt{x}}^1 \frac{1}{\sqrt{1+y^3}} \, dy \, dx$ $\quad 0 \le x \le 1,\ \sqrt{x} \le y \le 1$



$$= \int_0^1 \int_0^{y^2} \frac{1}{\sqrt{1+y^3}} \, dx \, dy$$

$$= \int_0^1 \left(\frac{x}{\sqrt{1+y^3}}\Big|_{x=0}^{x=y^2}\right) dy$$

$$= \int_0^1 \frac{y^2}{\sqrt{1+y^3}} \, dy$$

$$= \frac{2}{3}(1+y^3)^{1/2}\Big|_0^1 = \frac{2}{3}\sqrt{2} - \frac{2}{3}$$

38.) $\int_1^2 \int_1^y \frac{\ln x}{x}\,dx\,dy + \int_2^4 \int_{y/2}^2 \frac{\ln x}{x}\,dx\,dy \rightarrow$

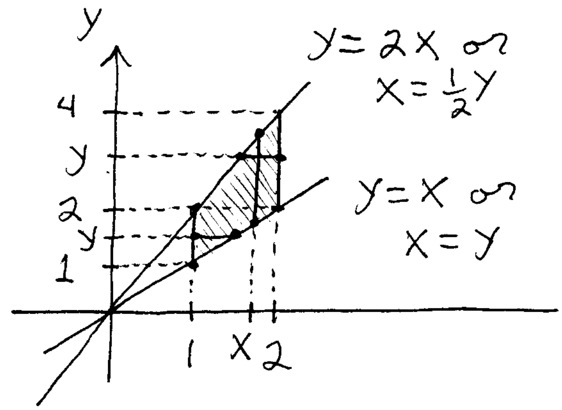
$1 \le y \le 2$ and $1 \le x \le y$;

$2 \le y \le 4$ and $\frac{y}{2} \le x \le 2$

~~then~~

$1 \le x \le 2$ and $x \le y \le 2x$

~~so that~~



$$\int_1^2 \int_1^y \frac{\ln x}{x}\,dx\,dy + \int_2^4 \int_{y/2}^2 \frac{\ln x}{x}\,dx\,dy$$

$$= \int_1^2 \int_x^{2x} \frac{\ln x}{x}\,dy\,dx$$

$$= \int_1^2 \left(y \cdot \frac{\ln x}{x} \Big|_{y=x}^{y=2x} \right) dx$$

$$= \int_1^2 (2\ln x - \ln x)\,dx$$

$$= \int_1^2 \ln x\,dx \qquad \left(\text{Let } u = \ln x,\ dv = dx \quad du = \frac{1}{x}dx,\ v = x \right)$$

$$= x\ln x \Big|_1^2 - \int_1^2 1\,dx$$

$$= 2\ln 2 - 1 \cdot \overset{0}{\cancel{\ln 1}} - x\Big|_1^2$$

$$= 2\ln 2 - (2-1)$$

$$= 2\ln 2 - 1 \ .$$