

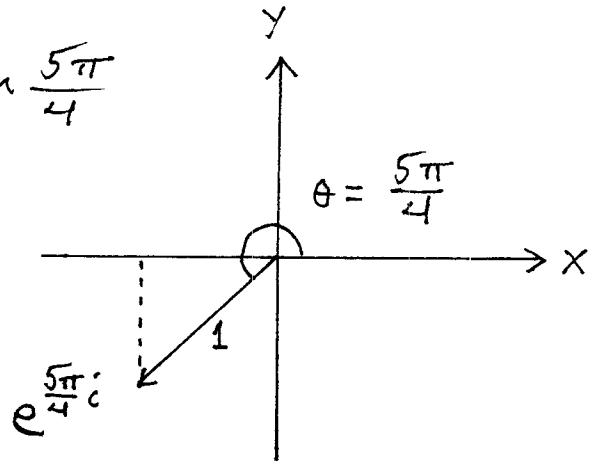
Section 11.7

$$1.) e^{\frac{5\pi}{4}i} = \cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4}$$

$$= -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i;$$

$$\operatorname{Re}(e^{\frac{5\pi}{4}i}) = -\frac{\sqrt{2}}{2}$$

$$\operatorname{Im}(e^{\frac{5\pi}{4}i}) = -\frac{\sqrt{2}}{2}.$$



$$6.) 2e^{\pi i} \cdot 3e^{-\frac{\pi}{3}i} = 6e^{\pi i + -\frac{\pi}{3}i}$$

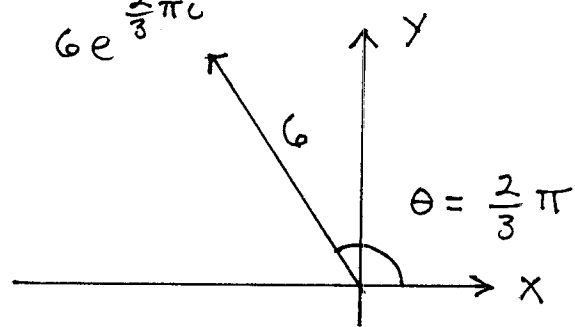
$$= 6 \cdot e^{\frac{2}{3}\pi i} = 6(\cos \frac{2}{3}\pi + i \sin \frac{2}{3}\pi)$$

$$= 6(-\frac{1}{2} + \frac{\sqrt{3}}{2}i)$$

$$= -3 + 3\sqrt{3}i;$$

$$\operatorname{Re}(6 \cdot e^{\frac{2}{3}\pi i}) = -3,$$

$$\operatorname{Im}(6 \cdot e^{\frac{2}{3}\pi i}) = 3\sqrt{3}.$$



$$9.) 5(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}) \cdot 3(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2})$$

$$= 15(\cos(\frac{\pi}{6} + \frac{\pi}{2}) + i \sin(\frac{\pi}{6} + \frac{\pi}{2}))$$

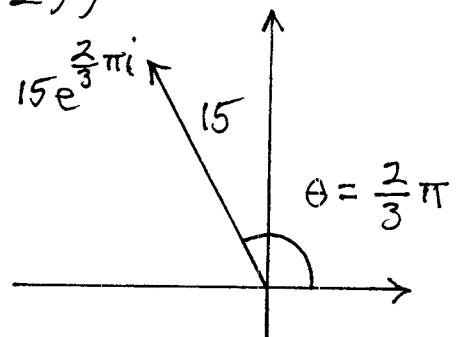
$$= 15(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3})$$

$$= 15(-\frac{1}{2} + \frac{\sqrt{3}}{2}i) = 15e^{\frac{2}{3}\pi i}$$

$$= -\frac{15}{2} + \frac{15\sqrt{3}}{2}i$$

$$\operatorname{Re}(15e^{\frac{2}{3}\pi i}) = -\frac{15}{2},$$

$$\operatorname{Im}(15e^{\frac{2}{3}\pi i}) = \frac{15\sqrt{3}}{2}.$$



$$\begin{aligned}
 17.) \quad e^z = 1 &\Rightarrow e^{a+bi} = 1 \Rightarrow \\
 e^a e^{bi} = 1 &\Rightarrow e^a (\cos b + i \sin b) = 1 \cdot (1 + i(0)) \Rightarrow \\
 e^a = 1 &\Rightarrow \boxed{a=0} \quad \text{and} \quad \boxed{b=2\pi(n)} \\
 &\text{for } n=0, \pm 1, \pm 2, \pm 3, \dots \Rightarrow \\
 z = a+bi &= 2\pi n i \quad \text{for } n=0, \pm 1, \pm 2, \pm 3, \dots
 \end{aligned}$$

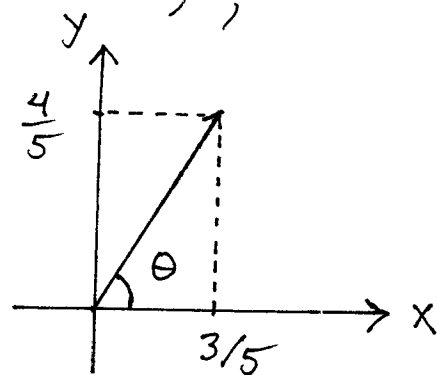
$$\begin{aligned}
 18.) \quad e^z = -1 &\Rightarrow e^{a+bi} = -1 \Rightarrow \\
 e^a e^{bi} = -1 &\Rightarrow e^a (\cos b + i \sin b) = 1 \cdot (-1 + i(0)) \Rightarrow \\
 e^a = 1 &\Rightarrow \boxed{a=0} \quad \text{and} \quad \boxed{b=\pi + 2\pi(n)} \\
 &\text{for } n=0, \pm 1, \pm 2, \pm 3, \dots \Rightarrow \\
 z = a+bi &= (\pi + 2\pi(n)) i \quad \text{for } n=0, \pm 1, \pm 2, \pm 3, \dots
 \end{aligned}$$

$$\begin{aligned}
 23.) \quad e^z = 3+4i &\Rightarrow e^{a+bi} = 3+4i \Rightarrow \\
 e^a e^{bi} = 5 \left(\frac{3}{5} + \frac{4}{5} i \right) &\Rightarrow \\
 e^a (\cos b + i \sin b) &= 5 (\cos \theta + i \sin \theta),
 \end{aligned}$$

$$\begin{aligned}
 &\text{where } \theta = \arctan\left(\frac{4}{3}\right); \\
 \Rightarrow e^a = 5 &\Rightarrow \boxed{a = \ln 5}
 \end{aligned}$$

$$\text{and } \boxed{b = \theta + 2\pi(n)}$$

$$\text{for } n=0, \pm 1, \pm 2, \pm 3, \dots$$



$$25.) e^{(n\theta)i} = \cos n\theta + i \sin n\theta \Rightarrow$$

$$\frac{e^{(n\theta)i}}{2^n} = \frac{\cos n\theta}{2^n} + i \frac{\sin n\theta}{2^n} \Rightarrow$$

$$\sum_{n=0}^{\infty} \frac{e^{(n\theta)i}}{2^n} = \sum_{n=0}^{\infty} \frac{\cos n\theta}{2^n} + i \sum_{n=0}^{\infty} \frac{\sin n\theta}{2^n} \Rightarrow$$

$$\sum_{n=0}^{\infty} \left(\frac{e^{\theta i}}{2}\right)^n = A + Bi \quad ; \text{ then}$$

$$\sum_{n=0}^{\infty} \left(\frac{e^{\theta i}}{2}\right)^n = 1 + \left(\frac{e^{\theta i}}{2}\right) + \left(\frac{e^{\theta i}}{2}\right)^2 + \left(\frac{e^{\theta i}}{2}\right)^3 + \dots$$

$$= \frac{1}{1 - \left(\frac{e^{\theta i}}{2}\right)} = \frac{2}{2 - e^{\theta i}} = \frac{2}{2 - \cos\theta - i \sin\theta}$$

$$= \frac{2}{(2 - \cos\theta) - i \sin\theta} \cdot \frac{(2 - \cos\theta) + i \sin\theta}{(2 - \cos\theta) + i \sin\theta}$$

$$= \frac{(4 - 2 \cos\theta) + i(2 \sin\theta)}{(2 - \cos\theta)^2 - (-1) \sin^2\theta}$$

$$= \frac{(4 - 2 \cos\theta) + i(2 \sin\theta)}{4 - 4 \cos\theta + \underbrace{\cos^2\theta + \sin^2\theta}_1}$$

$$= \left(\frac{4 - 2 \cos\theta}{5 - 4 \cos\theta}\right) + i \cdot \left(\frac{2 \sin\theta}{5 - 4 \cos\theta}\right) = C + Di ;$$

$$A = C \Rightarrow$$

$$\sum_{n=0}^{\infty} \frac{\cos n\theta}{2^n} = \frac{4 - 2 \cos\theta}{5 - 4 \cos\theta} .$$

$$26.) \quad e^{(n\theta)i} = \cos n\theta + i \sin n\theta \Rightarrow$$

$$\frac{e^{(n\theta)i}}{n!} = \frac{\cos n\theta}{n!} + i \frac{\sin n\theta}{n!} \Rightarrow$$

$$\sum_{n=0}^{\infty} \frac{(e^{\theta i})^n}{n!} = \sum_{n=0}^{\infty} \frac{\cos n\theta}{n!} + i \cdot \sum_{n=0}^{\infty} \frac{\sin n\theta}{n!}$$

$$= A + Bi ; \text{ then}$$

$$\sum_{n=0}^{\infty} \frac{(e^{\theta i})^n}{n!} = 1 + (e^{\theta i}) + \frac{(e^{\theta i})^2}{2!} + \frac{(e^{\theta i})^3}{3!} + \dots$$

$$= e^{e^{\theta i}} = e^{\cos\theta + i \sin\theta}$$

$$= e^{\cos\theta} e^{i(\sin\theta)}$$

$$= e^{\cos\theta} (\cos(\sin\theta) + i \sin(\sin\theta))$$

$$= e^{\cos\theta} \cdot \cos(\sin\theta) + i \cdot e^{\cos\theta} \cdot \sin(\sin\theta)$$

$$= C + Di ;$$

$$B = D \Rightarrow$$

$$\sum_{n=0}^{\infty} \frac{\sin n\theta}{n!} = e^{\cos\theta} \cdot \sin(\sin\theta)$$