Math 21C
Kouba
Practice Exam 1

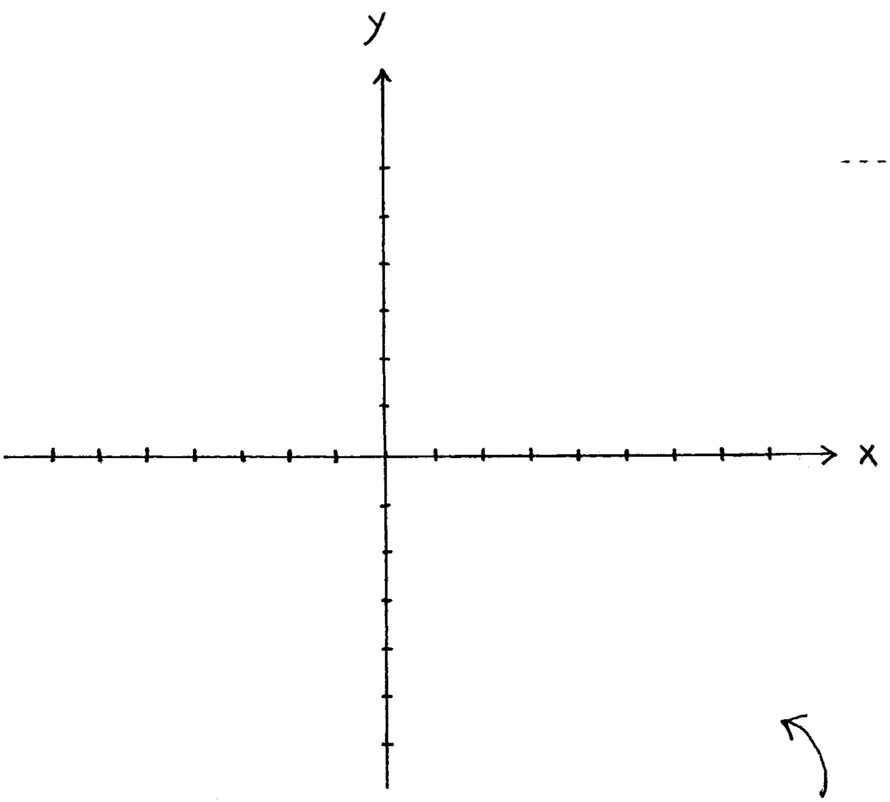
1.) (12 pts. each) Find $z_x$ and $z_y$ for each of the following. You need not simplify your answers.

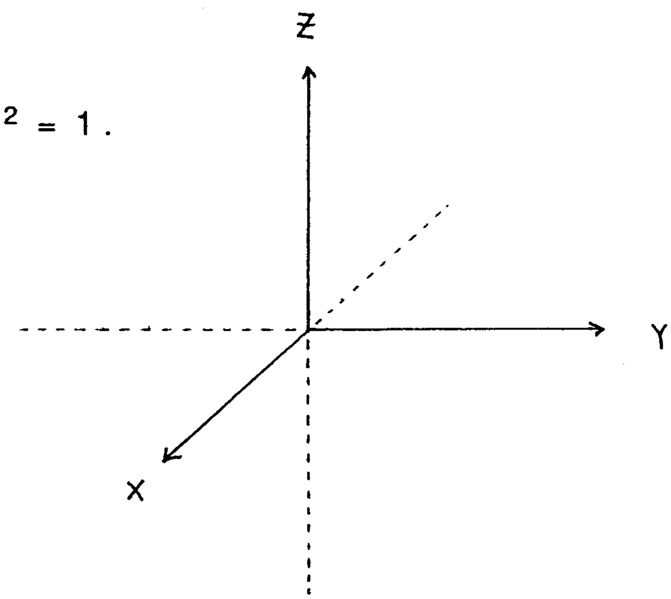
a.) $z = x y^3 + \tan(x - y)$

b.) $z = 7 + \ln\left\{ (e^{x^2}) + \arctan(\sin x) \right\}$

c.) $z = y^{x} \cos(x y)$

2.) (10 pts.) Neatly sketch the surface $z - x^2 - y^2 = 1$.



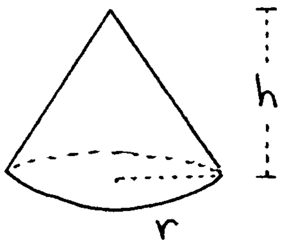




3.) (10 pts.) Consider the function $z = 4x \big/ (x^2 + y^2)$.

a.) Accurately sketch the level curves for the following values of $z$ : 4, 2, 1, 1/2, 0, −1/2, −1 .

b.) Describe in words what you think this surface looks like.

4.) (10 pts.) Assume that the height $h = h(t)$ and the radius $r = r(t)$ of the given cone are functions of time t. If the radius is increasing at the rate of 3 ft./sec. when $r = 2$ ft. and the height is decreasing at the rate of 2 ft./sec. when $h = 4$ ft., at what rate is the volume changing when $r = 2$ ft. and $h = 4$ ft. ? ( HINT : $V = 1/3\,\pi r^2 h$ . )




5.) (10 pts.) If the radius of the cone in problem 4.) is measured with a maximum possible error of 5% and the height is measured with a maximum possible error of 3%, what is the maximum possible error in measuring the volume of the cone ?

6.) (12 pts.) Verify that $f(x, y) = \ln(1 + x^r + y^r)$ , where r is a constant, satisfies the partial differential equation

$$f_{xy} + f_x f_y = 0 .$$

7.) (12 pts.) Assume that $w = f(x, y, z)$ and $f(u - t, t, u) = 0$ . Show that

$$f_y + f_z = 0 .$$

EXTRA CREDIT PROBLEM -- This problem is worth 10 points.

Find an equation for the plane tangent to the surface $z = x^2 + y^4$ at the point $(1,-1, 2)$. Also include a clear verification that the plane is tangent to the given surface at the specific point.