Practice

Math 21C Exam 1 Solutions

1.) a.) $z = xy^3 + \tan(x-y) \rightarrow$

$z_x = y^3 + \sec^2(x-y)\cdot(1)$ and

$z_y = 3xy^2 + \sec^2(x-y)\cdot(-1)$.

b.) $z = 7 + \ln\{ e^{x^2} + \arctan(\sin x)\} \rightarrow$

$z_x = \dfrac{1}{e^{x^2} + \arctan(\sin x)} \cdot \left[2xe^{x^2} + \dfrac{1}{1+\sin^2 x}\cdot \cos x\right]$ and

$z_y = 0$

c.) $z = y^{x\cos(xy)} \rightarrow$

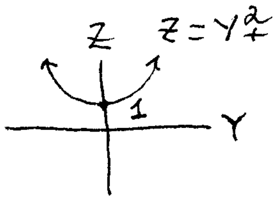
$z_x = y^{x\cos(xy)} \cdot \{x\cdot -\sin(xy)\cdot y + \cos(xy)\}\cdot \ln y$ and

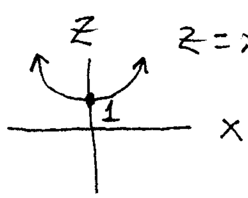
$\ln z = x\cos(xy)\cdot \ln y$ so

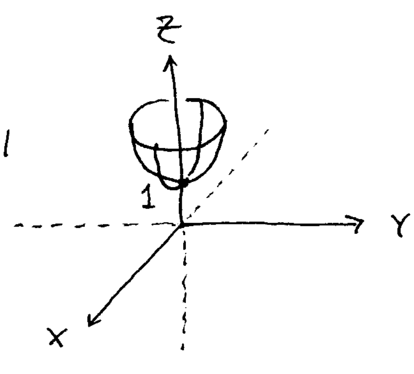
$\frac{1}{z} z_y = x\left[\cos(xy)\cdot \frac{1}{y} + -\sin(xy)\cdot x \cdot \ln y\right]$ or

$z_y = y^{x\cos(xy)} \cdot x\left[\cos(xy)\cdot\frac{1}{y} - \sin(xy)\cdot x\cdot \ln y\right]$.

2) $\left.\begin{array}{ll} x=0: & z = y^2+1 \\ y=0: & z = x^2+1 \end{array}\right\} z - x^2 - y^2 = 1$







level curves : $z = c \rightarrow x^2 + y^2 = c - 1$
(circles) for $c \geq 1$.

3.) a.)

$z = 4: \quad (x - \tfrac{1}{2})^2 + y^2 = (\tfrac{1}{2})^2$

$z = 2: \quad (x-1)^2 + y^2 = (1)^2$

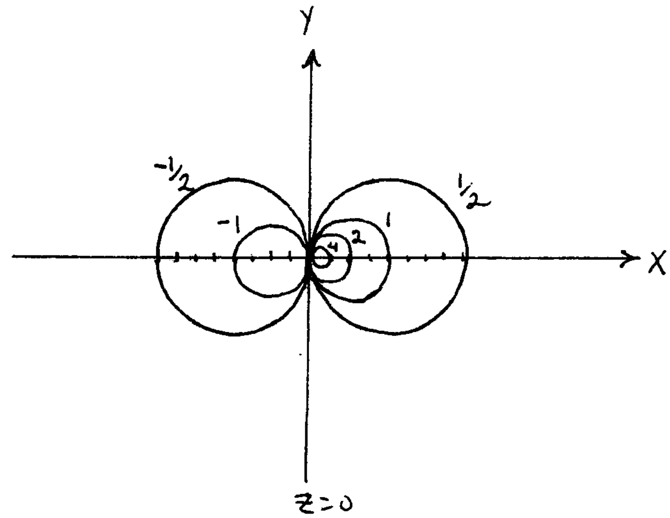
$z = 1: \quad (x-2)^2 + y^2 = (2)^2$

$z = \tfrac{1}{2}: \quad (x-4)^2 + y^2 = (4)^2$

$z = 0: \quad x = 0$

$z = -\tfrac{1}{2}: \quad (x+4)^2 + y^2 = (4)^2$

$z = -1: \quad (x+2)^2 + y^2 = (2)^2$



b.) symmetric, inverted "cones" tangent to the z-axis

4.) $V = \frac{1}{3}\pi r^2 h \rightarrow$

$$\frac{dV}{dt} = V_r \cdot \frac{dr}{dt} + V_h \cdot \frac{dh}{dt}$$

$$= \frac{2}{3}\pi r h \cdot \frac{dr}{dt} + \frac{1}{3}\pi r^2 \cdot \frac{dh}{dt}$$

$$= \frac{2}{3}\pi (2)(4)(3) + \frac{1}{3}\pi (2)^2 \cdot (-2) = \frac{40}{3}\pi \text{ ft.}^3/\text{sec.}$$

5.) $\frac{|\Delta r|}{r} \le 5\%$ and $\frac{|\Delta h|}{h} \le 3\%$ then

$$\frac{|\Delta V|}{V} \approx \frac{|dV|}{V} = \frac{|V_r \cdot \Delta r + V_h \cdot \Delta h|}{V}$$

$$\le \frac{\frac{2}{3}\pi r h |\Delta r|}{\frac{1}{3}\pi r^2 h} + \frac{\frac{1}{3}\pi r^2 |\Delta h|}{\frac{1}{3}\pi r^2 h}$$

$$= 2\frac{|\Delta r|}{r} + \frac{|\Delta h|}{h} = 2(5\%) + 3\% = 13\%$$

6.) $f(x,y) = \ln(1 + x^r + y^r) \rightarrow$

$$f_x = \frac{r x^{r-1}}{1 + x^r + y^r}, \quad f_y = \frac{r y^{r-1}}{1 + x^r + y^r}, \quad \text{and}$$

$$f_{xy} = \frac{-r x^{r-1} \cdot r y^{r-1}}{(1 + x^r + y^r)^2} \quad \text{so that}$$

$$f_{xy} + f_x f_y = \frac{-r^2 x^{r-1} y^{r-1}}{(1+x^r+y^r)^2} + \frac{-r^2 x^{r-1} y^{r-1}}{(1+x^r+y^r)^2} = 0 \ .$$

7.) $w = f(x,y,z)$ and $f(u-t, t, u) = 0$ so that

$x = u - t, \ y = t, \ z = u$ and

$$f_x \cdot \frac{\partial(x)}{\partial t} + f_y \cdot \frac{\partial(y)}{\partial t} + f_z \frac{\partial(z)}{\partial t} = \frac{\partial(0)}{\partial t} \rightarrow$$

$$-f_x + f_y + 0 = 0 \qquad \text{and}$$

$$f_x \cdot \frac{\partial(x)}{\partial u} + f_y \cdot \frac{\partial(y)}{\partial u} + f_z \cdot \frac{\partial(z)}{\partial u} = \frac{\partial(0)}{\partial u} \rightarrow$$

$$f_x + 0 + f_z = 0 \qquad \text{(add)} \rightarrow$$

$$f_y + f_z = 0 \ .$$

<u>Extra Credit</u> : $z = x^2 + y^4 \rightarrow z_x = 2x$, $z_y = 4y^3$ so at $(1,-1,2)$ $z_x = 2$ and $z_y = -4$; plane is

$$z = 2 + (2)(x-1) + (-4)(y - (-1)) \text{ or } \boxed{z = 2 + 2(x-1) - 4(y+1)};$$

clearly $(1,-1,2)$ lies on plane and plane is tangent since for plane $z_x = 2$, $z_y = -4$, same as for the surface.