Math 21C
Kouba
Practice Exam 2

1.)   Evaluate the following definite integrals.

a.) $\displaystyle\int_0^1 \int_0^{\sqrt{1-x^2}} e^{x^2+y^2} \, dy \, dx$

b.) $\displaystyle\int_{\pi/4}^{\pi} \int_{\pi/x}^{2} \int_0^{\cos(xy)} x \, dz \, dy \, dx$

2.)   A thin lamina lies in the triangular region with vertices $(0, 0)$, $(2, 2)$, and $(3, 2)$. Density at point (x, y) is $f(x, y) = x^2 + y$. Find its moment of inertia about the line $y = 2$. SET UP BUT DO NOT EVALUATE THE INTEGRAL(S).

3.)   Consider the solid region with vertices $(0, 0, 0)$, $(3, 0, 0)$, $(0, 3, 0)$, and $(0, 0, 3)$. Find its volume. SET UP BUT DO NOT EVALUATE THE INTEGRAL(S).

4.)   Assume that region R is described in polar coordinates by $a \le \theta \le b$ and $0 \le r \le f(\theta)$. Show that the area of region R is

$$\text{Area} = \int_a^b \frac{1}{2} \left[f(\theta) \right]^2 \, d\theta.$$

5.) Consider the solid region R above the xy-plane, inside the cylinder $(x - 1/2)^2 + y^2 = \frac{1}{4}$, and below the plane $z = y + 1$. Using cylindrical coordinates SET UP but DO NOT EVALUATE integrals for the y-coordinate of the centroid of the solid.

6.) The following triple integral represents the volume of a solid in three dimensional space. Sketch the solid and compute its volume.

$$\int_0^1 \int_0^{1-x} \int_{1-x-y}^1 1 \, dz \, dy \, dx$$

7.) Use spherical coordinates to compute the volume of a sphere of radius a.