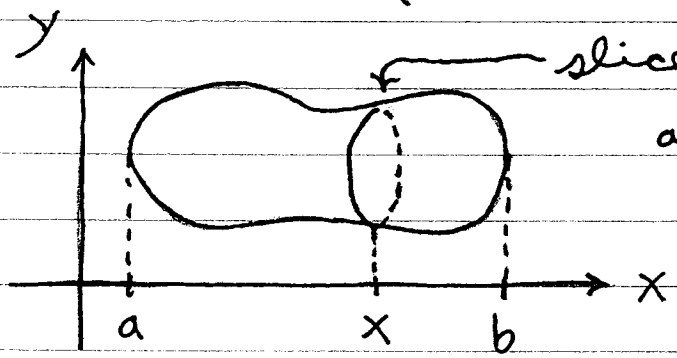
Evaluating $\iint_R f(P)\,dA$ Using Rectangular Coordinates

Recall: If a solid has a known cross-sectional area $A(x)$ if slice is made perpendicular to the x-axis at x, then the volume of the solid is

$$\text{Volume} = \int_a^b A(x)\,dx.$$

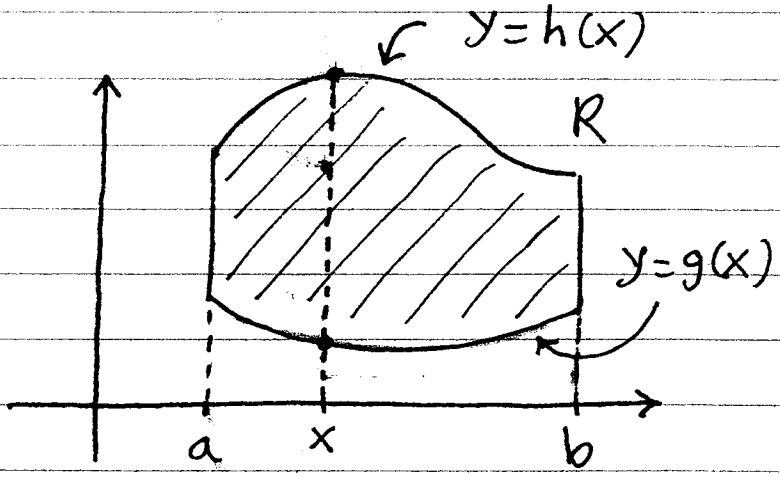


Let region R in the plane be given by

$$a \le x \le b$$

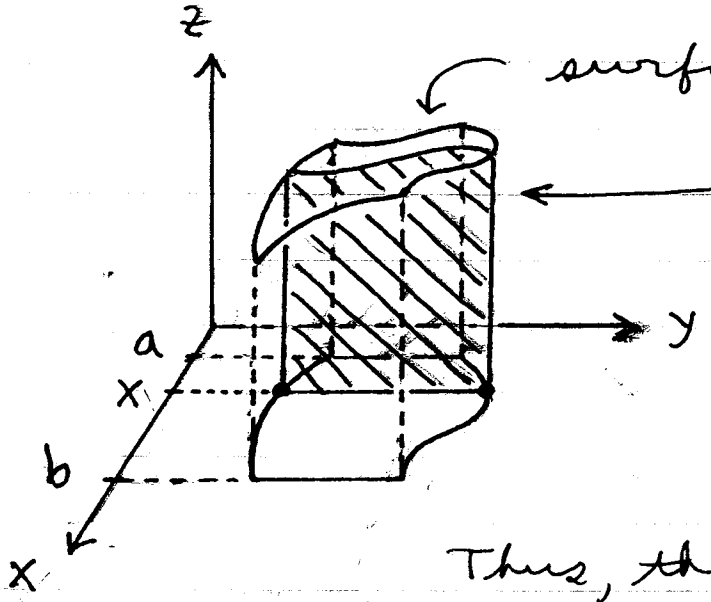
and

$$g(x) \le y \le h(x).$$



Let $z = f(x,y)$ be a function of two variables. We seek a method to evaluate

$$\iint_R f(P)\,dA.$$



$$A(x) = \int_{g(x)}^{h(x)} f(x,y)\, dy .$$

Thus, the volume of the solid is

$$\iint_R f(P)\, dA = \int_a^b A(x)\, dx$$

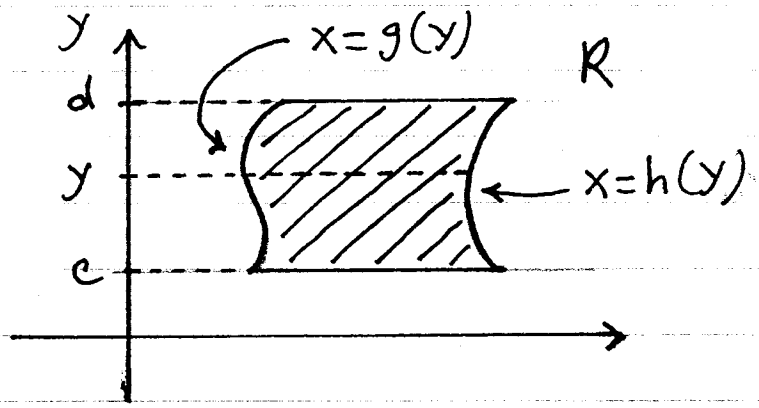
$$= \int_a^b \int_{g(x)}^{h(x)} f(x,y)\, dy\, dx .$$

Let region R in the plane be given by

$$c \le y \le d$$

and

$$g(y) \le x \le h(y) .$$



It follows analogously that

$$\iint_R f(P)\, dA = \int_c^d \int_{g(y)}^{h(y)} f(x,y)\, dx\, dy$$