Math 21C DHC
Kouba
Discussion Sheet 1

1.) Sketch the following surfaces in three-dimensional space.

a.) $x^2 + y^2 + z^2 = 4$
b.) $x^2 + y^2 = z^2 + 4$
c.) $x^2 = y^2 + z^2 + 4$

2.) Sketch the following two surfaces and their intersection in three-dimensional space. Also plot the projection of this intersection in the xy-plane :

$$z = x^2 + y^2 \quad \text{and} \quad z = \sqrt{20 - x^2 - y^2}$$

3.) a.) Plot the level curves for $z = \dfrac{y}{x}$ using the following values of z :
$0, \pm 1, \pm 2, \pm 3, \pm 1/2, \pm 10$

b.) Use part a.) to describe in words what the surface $z = \dfrac{y}{x}$ looks like.

4.) Evaluate the following limits or justify that the limit does not exist.

a.) $\displaystyle \lim_{(x,y)\to(1,-1)} \frac{x^3 + y^3}{x^2 - y^2}$
b.) $\displaystyle \lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^4 + 3y^2}$
c.) $\displaystyle \lim_{(x,y)\to(0,0)} \frac{x^3 + y^3}{x + y^3}$

5.) Consider the graph of $y = x$ in the xy-plane. Form a surface in three dimensional space by rotating the x-axis about this line. Determine a formula for this surface.

6.) Give an ϵ/δ proof of the fact that

a.) $\displaystyle \lim_{(x,y)\to(0,0)} (x^4 - y^4) = 0$.

b.) $\displaystyle \lim_{(x,y)\to(0,0)} \frac{xy}{x^2 + y^2 + 3} = 0$.