Math 21C DHC
Kouba
Discussion Sheet 2

1.) Show that $T = \dfrac{1}{\sqrt{x^2+y^2}}$ satisfies the equation $\dfrac{\partial^2 T}{\partial x^2} + \dfrac{\partial^2 T}{\partial y^2} = T^3$.

2.) Find z_x, z_y, z_{xx}, z_{yy}, and z_{xy} for $z = \ln(xy^2+3)$.

3.) Find a function $z = f(x,y)$ with the following partial derivatives or state that this is impossible :

$$z_x = e^{x^2y}\cos x + 2xye^{x^2y}\sin x + 2xy^3 + 1$$
$$z_y = x^2e^{x^2y}\sin x + 3x^2y^2 + 2ye^{y^2}.$$

4.) Assume that $u = f(x,y)$, $x = r\cos\theta$, and $y = r\sin\theta$. Compute $\dfrac{\partial u}{\partial r}$, $\dfrac{\partial u}{\partial \theta}$, and $\dfrac{\partial^2 u}{\partial \theta^2}$.

5.) Find an equation of the plane tangent to the surface $z = y^2 - x^2$ at the point $(2, 1, -3)$.

6.) Find the point on the plane $3x + 2y + z = 12$ which is nearest the origin.

7.) a.) Show that $(0,0)$ is a critical point for $z = x^4 - 2x^2y^2 + y^4$. Show that $(0,0)$ determines a minimum value.

b.) Show that $(0,0)$ is a critical point for $z = 2x^4 + 4x^3y + y^4$. Show that $(0,0)$ determines a saddle point.

c.) Show that $(0,0)$ is a critical point for $z = (y - x^2)(y - 2x^2)$. Show that $(0,0)$ determines a saddle point.

9.) A house in the shape of a rectangular box is to hold 10,000 cubic feet. The four walls admit heat at 5 units per minute per square foot. The roof admits heat at 3 units per minute per square foot. The floor admits heat at 2 units per minute per square foot. What should the dimensions (length, width, height) of the house be in order to minimize the rate (units per minute) at which heat enters ?