Math 21C (Spring 2003)
Kouba
Exam 1

Please PRINT your name here : _____

Please SIGN your name here : _____

Your Exam ID Number _____

1. PLEASE DO NOT TURN THIS PAGE UNTIL TOLD TO DO SO.

2. IT IS A VIOLATION OF THE UNIVERSITY HONOR CODE TO, IN ANY WAY, ASSIST ANOTHER PERSON IN THE COMPLETION OF THIS EXAM. PLEASE KEEP YOUR OWN WORK COVERED UP AS MUCH AS POSSIBLE DURING THE EXAM SO THAT OTHERS WILL NOT BE TEMPTED OR DISTRACTED. THANK YOU FOR YOUR COOPERATION.

3. YOU MAY USE A CALCULATOR ON THIS EXAM.

4. No notes, books, or classmates may be used as resources for this exam.

5. Read directions to each problem carefully. Show all work for full credit. In most cases, a correct answer with no supporting work will NOT receive full credit. What you write down and how you write it are the most important means of your getting a good score on this exam. Neatness and organization are also important.

6. You have until 9:52 a.m. sharp to finish the exam.

7. Make sure that you have 5 pages including the cover page..

1.) (12 pts.) Sketch the surfaces $z = x^2 + y^2$ and $z = 3 - 2x$ and their intersection on the same set of axes in three-dimensional space. On another set of axes sketch the projection of this intersection in the xy-plane.

2.) (8 pts.) Determine the radius and center of the sphere given by $x^2 + 2x + y^2 - 6y + z^2 = -6$.

3.) (10 pts.) Find an equation of the surface in three-dimensional space formed by revolving the equation $z = \ln x$ about the z-axis.

4.) Evaluate the following limits or determine that the limit does not exist.

a.) (8 pts.) $\lim_{(x,y)\to(3,-3)} \dfrac{x^4 - y^4}{x + y}$

b.) (8 pts.) $\lim_{(x,y)\to(0,0)} \dfrac{xy^2}{x^2 + y^4}$

5.) (8 pts.) Determine the domain and sketch (shade) this domain in the xy-plane : $f(x, y) = \sqrt{y^2 - x^2}$.

6.) (12 pts.) Show that $f(x, y) = \ln(x^2 + y^2)$ satisfies the equation $f_{xx} + f_{yy} = 0$.

7.) (10 pts.) Determine an equation of the plane tangent to the graph of $z = 2xy$ at the point $(1, -3, -6)$. HINT : Assume that an equation of the plane is $z = Ax + By + C$ and solve for the unknown constant A, B, and C.

8.) (14 pts.) Find and classify (relative maximum, relative minimum, or saddle point) the critical points for $z = x^3 + 3xy - y^3$.

9.) (10 pts.) Assume that $z = f(x, y)$, $x = r\cos\theta$, and $y = r\sin\theta$. Compute the second partial derivative $\dfrac{\partial^2 z}{\partial \theta^2}$. DO NOT SIMPLIFY your final answer.

The following EXTRA CREDIT PROBLEM is worth 12 points. This problem is OPTIONAL.

1.) Consider the graph of $y = x$ in the xy-plane. Find an equation of the surface in three-dimensional space formed by revolving the x-axis about this line.