Math 21C (Spring 2003)
Kouba
Exam 2

Please PRINT your name here : ____

Please SIGN your name here : ____

Your Exam ID Number ___________

1. PLEASE DO NOT TURN THIS PAGE UNTIL TOLD TO DO SO.

2. IT IS A VIOLATION OF THE UNIVERSITY HONOR CODE TO, IN ANY WAY, ASSIST ANOTHER PERSON IN THE COMPLETION OF THIS EXAM. PLEASE KEEP YOUR OWN WORK COVERED UP AS MUCH AS POSSIBLE DURING THE EXAM SO THAT OTHERS WILL NOT BE TEMPTED OR DISTRACTED. THANK YOU FOR YOUR COOPERATION.

3. YOU MAY USE A CALCULATOR ON THIS EXAM.

4. No notes, books, or classmates may be used as resources for this exam.

5. Read directions to each problem carefully. Show all work for full credit. In most cases, a correct answer with no supporting work will NOT receive full credit. What you write down and how you write it are the most important means of your getting a good score on this exam. Neatness and organization are also important.

6. You have until 9:52 a.m. sharp to finish the exam. PLEASE STOP WRITING IMMEDIATELY when time is called and close your exam.

7. Make sure that you have 5 pages including the cover page.

1.) (12 pts.) Assume that $z = f(x, y)$, $x = r^2 s$, and $y = 3r + 2s$. Compute the second partial derivative $\frac{\partial^2 z}{\partial r^2}$. DO NOT SIMPLIFY your final answer.

2.) (5 pts. each) Consider the flat triangular region R bounded by the graphs of $y = x$, $y = 3x$, and $x = 2$. Sketch the region and describe R using

a.) vertical cross sections.

b.) horizontal cross sections.

c.) polar coordinates.

3.) (12 pts. each) Evaluate each of the following double integrals.

a.) $\displaystyle\int_0^{\pi/2}\int_0^{x} 2\cos(x+y)\,dy\,dx$

b.) $\displaystyle\int_0^{2}\int_{y/2}^{1} e^{x^2}\,dx\,dy$

4.) (12 pts.) Consider the solid region R enclosed by the paraboloid $z = 4 - x^2 - y^2$ and the plane $z = 0$. Assume that the density at point $P = (x, y, z)$ is numerically equal to the distance from P to the xy-plane. SET UP BUT DO NOT EVALUATE a triple integral in rectangular coordinates which represents the *total mass* of R.

5.) (13 pts.) Set up and EVALUATE a triple integral (using your choice of coordinate system) representing the *volume* of a sphere of radius a.

6.) (12 pts.) Consider the solid region R enclosed by the cylinder $x^2 + y^2 = 1$, the plane $z = 0$, and the plane $z = y + 2$. Assume that the density at point $P = (x, y, z)$ is given by $f(x, y, z) = x^2 z$. SET UP BUT DO NOT EVALUATE a triple integral in cylindrical coordinates which represents the *moment of inertia* of R about the z-axis.

7.) (12 pts.) Consider the solid region R enclosed by the cone $z = \sqrt{x^2+y^2}$ and the hemisphere $z = \sqrt{8-x^2-y^2}$. SET UP BUT DO NOT EVALUATE a triple integral in spherical coordinates which represents the *average value* of function $f(x,y,z) = yz$ over region R.

Each of the following EXTRA CREDIT PROBLEMS is worth 10 points. These problem are OPTIONAL.

1.) Consider the tetrahedron with vertices $(0,0,0)$, $(0,0,1)$, $(0,1,1)$, and $(1,1,1)$. Describe this solid using

a.) cylindrical coordinates.

b.) spherical coordinates.