Math 21C (Spring 2003)
Kouba
Exam 3

Please PRINT your name here : ____

Please SIGN your name here : ____

Your Exam ID Number ________

1. PLEASE DO NOT TURN THIS PAGE UNTIL TOLD TO DO SO.

2. IT IS A VIOLATION OF THE UNIVERSITY HONOR CODE TO, IN ANY WAY, ASSIST ANOTHER PERSON IN THE COMPLETION OF THIS EXAM. PLEASE KEEP YOUR OWN WORK COVERED UP AS MUCH AS POSSIBLE DURING THE EXAM SO THAT OTHERS WILL NOT BE TEMPTED OR DISTRACTED. THANK YOU FOR YOUR COOPERATION.

3. YOU MAY USE A CALCULATOR ON THIS EXAM.

4. No notes, books, or classmates may be used as resources for this exam.

5. Read directions to each problem carefully. Show all work for full credit. In most cases, a correct answer with no supporting work will NOT receive full credit. What you write down and how you write it are the most important means of your getting a good score on this exam. Neatness and organization are also important.

6. You have until 9:52 a.m. sharp to finish the exam. PLEASE STOP WRITING IMMEDIATELY when time is called and close your exam.

7. Make sure that you have 5 pages including the cover page.

8. The following may be used on the exam.

$$(*)\ \int_1^{n+1} f(x)\,dx < \underbrace{f(1) + f(2) + \cdots + f(n)}_{S_n} < f(1) + \int_1^{n} f(x)\,dx$$

$$(*)(*)\ \int_{n+1}^{\infty} f(x)\,dx < \underbrace{f(n+1) + f(n+2) + f(n+3) + \cdots}_{R_n} < \int_n^{\infty} f(x)\,dx$$

1.) (8 pts. each) Determine whether each of the following series converges or diverges. Write clear and complete solutions including the name of the series test that you use and what your final answer is.

a.) $\displaystyle\sum_{n=1}^{\infty} \frac{2^{n+1}}{(n-1)!}$

b.) $\displaystyle\sum_{n=2}^{\infty} \frac{n+1}{5n+2}$

c.) $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{5n^3+2}$

d.) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

e.) $\sum_{n=1}^{\infty} \left(\frac{n+2}{2n+3} \right)^n$

f.) $\sum_{n=2}^{\infty} (-1)^{n+1} \frac{1}{n(n^2+n+1)}$

g.) $\sum_{n=1}^{\infty} \left(\frac{1}{n+1} - \frac{1}{n+2} \right)$

h.) $\sum_{n=2}^{\infty} \frac{\ln n}{n^2}$

2.) (8 pts.) Find the exact value of the following convergent geometric series :

$$\sum_{n=1}^{\infty} \frac{3^{n+2}}{4^{n-1}}$$

3.) (8 pts.) Use a geometric series to convert the decimal number $0.09999\cdots$ to a fraction.

4.) (5 pts. each) The alternating series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n}$ converges.

a.) If $S_5 = \sum_{i=1}^{5}(-1)^{i+1}\frac{1}{i}$ is used to estimate the exact value of the series, is S_5 an over estimate or an under estimate ? Why ?

b.) What should n be in order that the partial sum $S_n = \sum_{i=1}^{n}(-1)^{i+1}\frac{1}{i}$ estimate the exact value of the series with absolute error at most 0.001 ?

5.) (10 pts.) The series $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ converges. What should n be in order that the partial sum $S_n = \sum_{i=1}^{n} \frac{1}{i^2+1}$ estimate the exact value of the series with error at most 0.001 ?

Each of the following EXTRA CREDIT PROBLEMS is worth 10 points. These problem are OPTIONAL.

1.) Determine whether each of the following series converges or diverges. Write clear and complete solutions including the name of the series test that you use and what your final answer is.

a.) $\sum_{n=1}^{\infty} \left(\frac{n^2}{n^2+1}\right)^n$

b.) $\sum_{n=2}^{\infty} \frac{n^{n+1/n}}{(n+1/n)^n}$