Math 21C Kouba Discussion Sheet 9

1.) Compute the derivative of $f(x,y) = x^2 + xy$ at the point P = (1,-1) in the direction of vector $\overrightarrow{A} = \overrightarrow{i} - 2\overrightarrow{j}$.

2.) Compute the derivative of $f(x, y, z) = x - y^2 + z^3$ at the point P = (2, 0, -1) in the direction of vector $\overrightarrow{A} = \overrightarrow{i} - \overrightarrow{j} + \overrightarrow{k}$.

3.) Consider the function $f(x,y) = xy^3$ and the point P = (2,1). Determine all unit vectors \overrightarrow{u} so that $D_{\overrightarrow{u}}f(2,1)$ is

- a.) as large as possible.
- b.) as small as possible.
- c.) equal to zero.
- d.) equal to 1.

4.) Consider the surface given by $x^2 + 2y^2 + 3z^2 = 3$ and the point P = (1, -1, 0) on the surface. Find equations for

- a.) the plane tangent to the surface at point P.
- b.) the line normal (perpendicular) to the surface at point P.

5.) Consider the surface (hyperbolic paraboloid or saddle) given by $f(x,y) = 3x^2 - 2y^2 + 5$ and the point P = (2,3,-1) on the surface. Find equations for

- a.) the plane tangent to the surface at point P.
- b.) the line normal (perpendicular) to the surface at point P.

6.) Consider the function $f(x, y) = xe^{xy}$ and the point P = (0, 1). Use a differential to estimate the change in the values of f if

- a.) point P moves a distance of ds = 0.15 in the direction of vector $\overrightarrow{A} = 3 \overrightarrow{i} 4 \overrightarrow{j}$.
- b.) point P moves in a straight line to point Q = (1, 0).

7.) Consider the function $f(x, y, z) = xy^2 + yz - x^3z$ and the point P = (1, -1, 2). Use a differential to estimate the change in the values of f if point P moves a distance of ds = 0.2 in the direction of vector $\overrightarrow{A} = -\overrightarrow{i} - 2\overrightarrow{j} + 2\overrightarrow{k}$.

8.) Consider the function given by $f(x,y) = xy^2 - x^2y$ and the point P = (1,-1). Compute

- a.) the exact change of f and
- b.) use a differential to estimate the exact change of f

if point P moves in a straight line to point Q = (1.5, -0.7).

9.) Consider the function given by $f(x,y) = \ln(3x + 4y^2)$ and the point P = (5,2). Compute

- a.) the exact change of f and
- b.) use a differential to estimate the exact change of f

if point P moves a distance of ds = 1.4 in the direction of vector $\overrightarrow{A} = 5 \overrightarrow{i} + 12 \overrightarrow{j}$.

"An education isn't how much you have committed to memory, or even how much you know. It's being able to differentiate between what you know and what you don't." - Anatole France