Math 21D
Kouba
Discussion Sheet 1

1.) Let R be the region bounded by the graphs of $y = x^2$ and $y = 3x$.
   a.) Describe R using vertical cross-sections.
   b.) Describe R using horizontal cross-sections.

2.) Let R be the region inside the circle of radius 5 centered at $(3, 4)$ and to the right of the line $x = 3$.
   a.) Describe R using vertical cross-sections.
   b.) Describe R using horizontal cross-sections.

3.) Let R be the triangular region with vertices $(0, 0)$, $(2, 0)$, and $(3, 2)$.
   a.) Describe R using vertical cross-sections.
   b.) Describe R using horizontal cross-sections.

4.) Let R be the region bounded by the graphs of $x = y^2$ and $x = y + 2$.
   a.) Describe R using vertical cross-sections.
   b.) Describe R using horizontal cross-sections.

5.) Sketch each of the following regions described in two-dimensional space.
   a.) $0 \le x \le 3$, $2 \le y \le 4$ b.) $0 \le x \le 3$, $\sqrt{x} \le y \le x + 1$
   c.) $1 \le x \le 3$, $0 \le y \le \ln x$ d.) $0 \le y \le \ln 3$, $e^y \le x \le 3$
   e.) $0 \le y \le 1$, $0 \le x \le \arcsin y$

6.) Evaluate the following double integrals.

a.) $\displaystyle\int_0^1 \int_{x^2}^{x} xy^2 \, dy \, dx$ b.) $\displaystyle\int_{\pi/2}^{\pi} \int_0^{x^2} (1/x) \cos(y/x) \, dy \, dx$

(Beware of the next two.)

c.) $\displaystyle\int_0^1 \int_{4x}^{4} e^{-y^2} \, dy \, dx$ d.) $\displaystyle\int_0^2 \int_{y/2}^{1} \cos(x^2) \, dx \, dy$

7.) Consider the tetrahedron with vertices $(0, 0, 0)$, $(4, 0, 0)$, $(0, 3, 0)$, and $(0, 0, 2)$.
   a.) It's top surface is a plane. Find an equation for this plane.
   b.) Set up but do not evaluate a double integral which represents the volume of the tetrahedron.

8.) Sketch the solid in 3D-Space whose volume is given by the following double integrals.

a.) $\displaystyle\int_0^3 \int_0^2 4 \, dy \, dx$

b.) $\int_0^2 \int_0^{4-2x} (6 - 2x - (3/2)y)\ dy\, dx$

c.) $\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} (7 - x^2 - y^2)\ dy\, dx$

THE FOLLOWING PROBLEM IS FOR RECREATIONAL PURPOSES ONLY.

9.) A snail is at the bottom of a well which is 100 feet deep. Each day the snail climbs up 6 feet and down 4 feet. In how many days will the snail reach the top of the well ?