Math 21D
Kouba
Discussion Sheet 5

1.) Consider the mapping F given by $F(u,v) = (3u - 2v, u + v) = (x, y)$. Let R be the rectangle and its interior in the uv-plane with vertices $(0,0)$, $(2,0)$, $(2,3)$, and $(0,3)$.
a.) Find the image S of R under F and the area of S.
b.) Find a mapping G which maps S to R.

2.) Redo problem 1.) where R is the triangle and its interior with vertices $(0,0)$, $(-2,3)$, and $(2,0)$.

3.) Consider the mapping F given by $F(u,v,w) = (u - v + 2w, 2u + v - w, 3u + 2v + w) = (x,y,z)$. Let R be the rectangle box and its interior in the uvw-space with vertices $(0,0,0)$, $(2,0,0)$, $(2,3,0)$, $(0,3,0)$, $(0,0,4)$, $(2,0,4)$, $(2,3,4)$, $(0,3,4)$.
a.) Find the image S of R under F and the volume of S.
b.) Find a mapping G which maps S to R.

4.) Plot the curve C determined by each vector function.
a.) $\vec{r}(t) = e^t\ \vec{i} + e^{3t}\ \vec{j}$ for $-1 \le t \le 1$
b.) $\vec{r}(t) = \cos t\ \vec{i} + \sin t\ \vec{j}$ for $0 \le t \le 2\pi$
c.) $\vec{r}(t) = \sqrt{t}\cos t\ \vec{i} + \sqrt{t}\sin t\ \vec{j}$ for $0 \le t \le 4\pi$
d.) $\vec{r}(t) = 2t\ \vec{i} + 3t\ \vec{j} + 4t\ \vec{k}$ for $0 \le t \le 2$
e.) $\vec{r}(t) = \sin t\ \vec{i} + \cos t\ \vec{j} + t\ \vec{k}$ for $0 \le t \le 4\pi$

5.) Assume that the motion of a particle along path C is determined by the position function $\vec{r}(t) = f(t)\ \vec{i} + g(t)\ \vec{j} + h(t)\ \vec{k}$. We know that the speed of motion at time t is $\left|\vec{v}(t)\right| = \dfrac{ds}{dt} = \sqrt{(f'(t))^2 + (g'(t))^2 + (h'(t))^2}$. Show that the acceleration of motion at time t is given by $a(t) = \dfrac{\vec{v}(t)\cdot\vec{a}(t)}{\left|\vec{v}(t)\right|}$.

6.) Assume that the path C of a bird in flight is determined by the vector function $\vec{r}(t) = t\ \vec{i} + t^2\ \vec{j} + 2t\ \vec{k}$ for $t \ge 0$. Find the bird's position vector, velocity vector, speed, acceleration vector, and acceleration at time
a.) $t = 0$.
a.) $t = 1$.
a.) $t = 2$.

7.) The position of a bicyclist is determined by the vector function $\vec{r}(t) = (3t)\ \vec{i} + (3\sin t)\ \vec{j}$ for $0 \le t \le 2\pi$. Determine the bicyclist's maximum speed.

8.) Find vector function $\vec{r}(t)$ if $\vec{r}''(t) = \vec{i} + t\,\vec{j} + \cos 2t\,\vec{k}$, $\vec{r}'(0) = \vec{i} + \vec{j} + \vec{k}$, and $\vec{r}(0) = 2\vec{i} - \vec{j} - \vec{k}$.

9.) A super ball is projected at an angle of 75^o with initial speed 100 $m./sec.$

a.) How high does the ball go ?

b.) How long is the ball in the air ?

c.) How far downrange does the ball travel ?

10.) A ball bearing is projected at an angle of 60^o and lands 500 feet downrange. What was the ball bearing's initial speed ?

11.) A kiwi is projected at an angle of α degrees with an initial speed of 100 $m./sec.$ If it lands 200 meters downrange, what is α ?

12.) Assume that $\vec{u}(t) = a(t)\,\vec{i} + b(t)\,\vec{j} + c(t)\,\vec{k}$, $\vec{v}(t) = f(t)\,\vec{i} + g(t)\,\vec{j} + h(t)\,\vec{k}$, and $y = k(t)$.

a.) (Dot Product Rule) Prove that $D\{\vec{u}(t) \cdot \vec{v}(t)\} = \vec{u}(t) \cdot \vec{v}'(t) + \vec{u}'(t) \cdot \vec{v}(t)$.

b.) (Chain Rule) Prove that $D\{\vec{u}(k(t))\} = \vec{u}'(g(t))k'(t)$

THE FOLLOWING PROBLEM IS FOR RECREATIONAL PURPOSES ONLY.

13.) Find the limit of the following sequence of numbers :

$$2,\ 2 - \frac{1}{2}\ ,\ 2 - \frac{1}{2 - \frac{1}{2}}\ ,\ 2 - \frac{1}{2 - \frac{1}{2 - \frac{1}{2}}}\ ,\ 2 - \frac{1}{2 - \frac{1}{2 - \frac{1}{2 - \frac{1}{2}}}}\ \cdots$$