Math 21D
Kouba
Discussion Sheet 6

1.) Show that the curve plotted by projectile motion,
$\vec{r}(t) = (|\vec{v}_0| \cos\alpha \cdot t)\vec{i} + (|\vec{v}_0| \sin\alpha \cdot t - (1/2)gt^2)\vec{j}$, is a parabola in the xy-plane.

2.) Show that the maximum downrange distance for projectile motion with a given initial speed $|\vec{v}_0|$ occurs when $\alpha = 45^o$ and is $x = \dfrac{|\vec{v}_0|^2}{g}$ (Hint: See formula III on Projectile Motion Handout.).

3.) Consider path C plotted by vector function $\vec{r}(t) = t\,\vec{i} + \sqrt{t}\,\vec{j}$ for $0 \le t \le 9$.

a.) Sketch C.
b.) Find $\vec{v}(t)$, $\vec{T}(t)$, $\vec{N}(t)$, and $\vec{a}(t)$.
c.) Plot $\vec{r}(1)$, $\vec{v}(1)$, $\vec{T}(1)$, $\vec{N}(1)$, and $\vec{a}(1)$. Also compute the speed and acceleration of motion when $t = 1$.

4.) Let $\vec{r}(t) = (12\sin t)\,\vec{i} + (-12\cos t)\,\vec{j} + (5t)\,\vec{k}$ be a position vector function.

a.) Determine the position vector when
i.) $t = 0$ ii.) $t = 7\pi/6$
b.) Write t as a function of arc length s. Write $\vec{r}(t)$ as a function of arc length s, i.e., write $\vec{r}(t) = \vec{r}(t(s))$.
c.) Determine the position vector when
i.) $s = 0$ ii.) $s = 39\pi$

5.) Determine the length of path C determined $\vec{r}(t) = (\cos^3 t)\,\vec{i} + (\sin^3 t)\,\vec{j}$ for $0 \le t \le 2\pi$.

6.) Evaluate the following line integrals.

a.) $\displaystyle\int_C 2x\ ds$, $\vec{r}(t) = (1/2)t^2\,\vec{i} + (1/4)t^4\,\vec{j}$ for $0 \le t \le 2$

b.) $\displaystyle\int_C \sqrt{x^2 + z^2}\ ds$, $\vec{r}(t) = (2\cos t)\,\vec{j} + (2\sin t)\,\vec{k}$ for $\pi \le t \le 2\pi$

c.) $\displaystyle\int_C 3xyz\ ds$, $\vec{r}(t) = t\,\vec{i} + 2t\,\vec{i} - t\,\vec{j}$ for $0 \le t \le 4$

7.) A spring lies on the path determined by $\vec{r}(t) = (\sin t)\ \vec{\imath} + (\cos t)\ \vec{\jmath} + (2/3)t^{3/2}\ \vec{k}$ for $0 \le t \le 4\pi$. Sketch the wire and find it's length.

8.) Find the area of the vertical wall sitting on the xy-plane on the line $y = 2x$ from $x = 0$ to $x = 4$, if the height of the wall at the point (x, y) is xy^2.

9.) A wire lies on the path determined by the helix $\vec{r}(t) = \cos t\ \vec{\imath} + \sin t\ \vec{\jmath} + t\ \vec{k}$ for $0 \le t \le 2\pi$. It's density at point (x, y, z) is given by $\delta(x, y, z) = xy + z + 3$. Compute the

a.) length of the wire.
b.) mass of the wire.
c.) x-coordinate for its center of mass.
d.) z-coordinate for its centroid.
e.) moment of the wire relative to the plane $y = 1$.
f.) moment of inertia of the wire about
  i.) the origin ii.) the z-axis

THE FOLLOWING PROBLEM IS FOR RECREATIONAL PURPOSES ONLY.

10.) Divide the following figure into 4 parts each of the same size and shape.