Math 21D
Kouba
Discussion Sheet 7

1.) Find the work done by each force field $\vec{F}$ along the given path C.

a.) $\vec{F}(x,y) = (x^2y)\ \vec{i} + (y)\ \vec{j}$
i.) C : graph of $y = x^2$ for $0 \le x \le 2$
ii.) C : graph of $y = 2x$ for $0 \le x \le 2$

b.) $\vec{F}(x,y) = (x)\ \vec{i} + (y)\ \vec{j}$ and C : $\vec{r}(t) = (3\cos t)\ \vec{i} + (4\sin t)\ \vec{j}$ for $0 \le t \le 2\pi$

c.) $\vec{F}(x,y) = (y)\ \vec{i} + (yz)\ \vec{k}$
i.) C : $\vec{r}(t) = (\cos t)\ \vec{i} + (\sin t)\ \vec{j} + (3t)\ \vec{k}$ for $0 \le t \le 2\pi$
ii.) C : line segment from point $(0,0,0)$ to $0(3,-1,2)$

2.) Find the flux of each vector field $\vec{F}$ across the given closed path C.

a.) $\vec{F}(x,y) = (x^2)\ \vec{i} + (y^2)\ \vec{j}$ and C : circle $x^2 + y^2 = 1$

b.) $\vec{F}(x,y) = (xy)\ \vec{i} + (x^2)\ \vec{j}$ and C : ellipse $\left(\frac{x}{3}\right)^2 + \left(\frac{y}{4}\right)^2 = 1$

c.) $\vec{F}(x,y) = (x)\ \vec{i} + (y^2)\ \vec{j}$ and C : the parabola $y = x^2$ from point $(0,0)$ to $(1,1)$, the line $y = 1$ from point $(1,1)$ to $(0,1)$, and the line $x = 0$ from point $(0,1)$ to $(0,0)$

3.) Find a scalar function f whose gradient field $\vec{\nabla} f$ is given.

a.) $\vec{\nabla} f(x,y) = 3\ \vec{i} - 4\ \vec{j}$

b.) $\vec{\nabla} f(x,y) = (y^2)\ \vec{i} + (2xy)\ \vec{j}$

c.) $\vec{\nabla} f(x,y) = (x^2ye^{xy} + 2xe^{xy})\ \vec{i} + (x^3e^{xy})\ \vec{j}$

d.) $\vec{\nabla} f(x,y,z) = (3x)\ \vec{i} + (4y)\ \vec{j} + (5z)\ \vec{k}$

e.) $\vec{\nabla} f(x,y,z) = (yz)\ \vec{i} + (xz)\ \vec{j} + (xy)\ \vec{k}$

f.) $\vec{\nabla} f(x,y,z) = \dfrac{2x}{x^2+y^2+z^2}\ \vec{i} + \dfrac{2y}{x^2+y^2+z^2}\ \vec{j} + \dfrac{2z}{x^2+y^2+z^2}\ \vec{k}$

g.) $\vec{\nabla} f(x,y) = (xy^2ze^{xz} + y^2e^{xz})\ \vec{i} + (2xye^{xz})\ \vec{j} + (x^2y^2e^{xz})\ \vec{k}$

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"In mathematics, you don't understand things. You just get used to them." – Johann von Neumann, a famous Hungarian mathematician