Math 21D
Kouba
Discussion Sheet 8

1.) Show that each vector field is conservative. Then evaluate the work integral $\int_C \vec{F} \cdot \vec{T}\, ds$ for each along the given path C.

a.) $\vec{F}(x,y) = (2xy)\,\vec{\imath} + (x^2+y^3)\,\vec{\jmath}$, where C : curve $y = x^3(x-1)^2$ for $-1 \le x \le 2$

b.) $\vec{F}(x,y) = (\sin y)\,\vec{\imath} + (x\cos y + 3)\,\vec{\jmath}$, where C : ellipse $\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$

c.) $\vec{F}(x,y) = (2x)\,\vec{\imath} + (2yz^2)\,\vec{\jmath} + (2y^2z)\,\vec{k}$, where C : any path from $(0,0,0)$ to $(2,3,4)$

2.) Use Green's Theorems (Theorem 1, 2, or 3 from class) to evaluate each line integral.

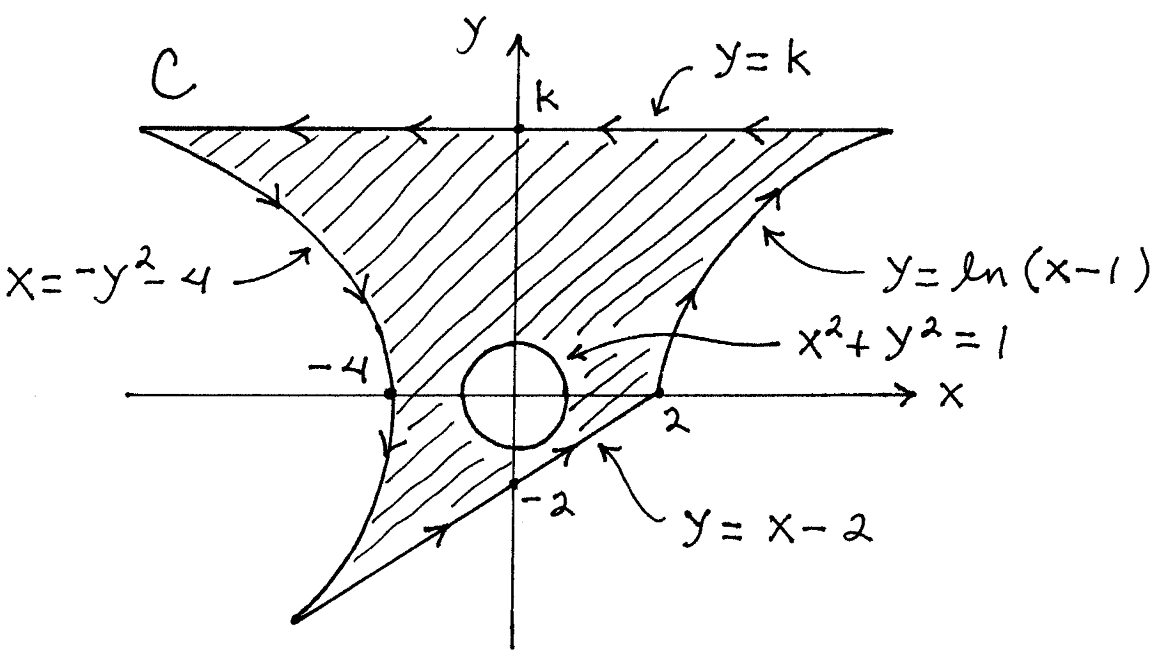
a.) $\int_C \vec{F} \cdot \vec{n}\, ds$, where $\vec{F}(x,y) = (3x)\,\vec{\imath} + (2y)\,\vec{\jmath}$ and C : circle $x^2+y^2=1$

b.) $\int_C (xy)dy - (x^2y)dx$, where C : rectangle with vertices $(0,0)$, $(3,0)$, $(3,2)$, and $(0,2)$

c.) $\int_C \vec{F} \cdot \vec{T}\, ds$, where $\vec{F}(x,y) = (\cos(x+y))\,\vec{\imath} + (\sin(x+y))\,\vec{\jmath}$ and C : triangle with vertices $(0,0)$, $(3,0)$, and $(0,4)$

d.) $\int_C (xy)dx + (e^x)dy$, where C : line segment joining $(0,0)$ to $(2,0)$, then the curve $y = 2x - x^2$ from $(2,0)$ to $((0,0)$

e.) $\int_C \vec{F} \cdot \vec{n}\, ds$, where $\vec{F}(x,y) = (x-y)\,\vec{\imath} + (x^2-2y)\,\vec{\jmath}$ and C is given in the diagram below. Assume that the top edge of path C is $y=k$, an unknown constant greater than 1, and that the area of the shaded region is 10: (HINT: Use Green's Theorem 3.)



3.) Use the fact that the area of region R enclosed by loop C is given by

$$\text{Area of } R = (1/2) \int_C (x)dy - (y)dx$$

to find the area inside the ellipse $\left(\frac{x}{4}\right)^2 + \left(\frac{y}{5}\right)^2 = 1$.

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"He who is not courageous enough to take risks will accomplish nothing in life." – Muhammad Ali, former world heavyweight boxing champion