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PROBLEM 1: Compute the area of the enclosed region bounded by the graphs of the equations $y = x$, $y = 2x$, and $x = 4$.

SOLUTION 1: Compute the area of the enclosed region bounded by the graphs of the equations $y = x$, $y = 2x$ and $x = 4$. Begin by finding the points of intersection of the two graphs. From $y = x$ and $y = 2x$ we get that

$x = 2x \quad \longrightarrow$

$-x = 0 \quad \longrightarrow$

$-x = 0 \quad \longrightarrow$

$x = 0 \quad \longrightarrow$

Using vertical cross-sections we get that

$$\text{AREA} = \int_0^4 (top \ - \ bottom) \ dx$$

$$= \int_0^4 (2x - x) \ dx$$

$$= \int_0^4 x \ dx$$

$$= x^2 2 \Big|_0^4$$

$$= 4^2 2 - 0^2 2$$

$$= 8 - 0$$

$$= 8$$

PROBLEM 2: Compute the area of the enclosed region bounded by the graphs of the equations $y = x^2$ and $y = x + 2$.

SOLUTION 2: Compute the area of the enclosed region bounded by the graphs of the equations $y = x^2$ and $y = x + 2$. Begin by finding the points of intersection of the two graphs. From $y = x^2$ and $y = x + 2$ we get that

$x^2 = x + 2 \quad \longrightarrow$

$x^2 - x - 2 = 0 \quad \longrightarrow$

$(x - 2)(x + 1) = 0 \quad \longrightarrow$

$x = 2 \ \text{ or } \ x = -1$

Using vertical cross-sections we get that

$$\text{AREA} = \int_{-1}^2 (top \ - \ bottom) \ dx$$

$$= \int_{-1}^{2} ((x+2) - x^2)\ dx$$

$$= \Big(\frac{x^2}{2} + 2x - \frac{x^3}{3} \Big) \Big|_{-1}^{2}$$

$$= \Big(\frac{2^2}{2} + 2(2) - \frac{2^3}{3} \Big) - \Big(\frac{(-1)^2}{2} + 2(-1) - \frac{(-1)^3}{3} \Big)$$

$$= \Big(2 + 4 - \frac{8}{3} \Big) - \Big(\frac{1}{2} - 2 + \frac{1}{3} \Big)$$

$$= \Big(6 - \frac{8}{3} \Big) - \Big(\frac{3}{6} - \frac{12}{6} + \frac{2}{6} \Big)$$

$$= \Big(\frac{36}{6} - \frac{16}{6} \Big) - \Big(\frac{-7}{6} \Big)$$

$$= \frac{20}{6} + \frac{7}{6}$$

$$= \frac{27}{6}\ .$$

PROBLEM 3: Compute the area of the enclosed region bounded by the graphs of the equations $y = e^x, y = e^{-x}$, and $x = \ln 3$.

SOLUTION 3: Compute the area of the enclosed region bounded by the graphs of the equations $y = e^x$, $y = e^{-x}$, and $x = \ln 3$. Begin by finding the points of intersection of the two graphs. From $y = e^x$ and $y = e^{-x}$ we get that

$$e^x = e^{-x} \quad \longrightarrow$$

$$e^{2x} = 1 \quad \longrightarrow$$

$$x = 0 \quad \longrightarrow$$

Using vertical cross-sections we get that

$$\text{AREA} = \int_0^{\ln 3} (top\ -\ bottom)\ dx$$

$$= \int_0^{\ln 3} (e^x - e^{-x})\ dx$$

$$= \Big(e^x - (-e^{-x}) \Big) \Big|_0^{\ln 3}$$

$$= \Big(e^x + e^{-x} \Big) \Big|_0^{\ln 3}$$

$$= \Big(e^{\ln 3} + e^{-\ln 3} \Big) - \Big(e^0 + e^{-0} \Big)$$

$$= \Big(3 + 13 \Big) - \Big(1 + 1 \Big)$$

$$= \Big(103 \Big) - \Big(2 \Big)$$

$= 103 - 63$

$= 43$

PROBLEM 4: Compute the area of the enclosed region bounded by the graphs of the equations $y = x^2$ and $y = -x^2 + 4x + 6$.

SOLUTION 4: Compute the area of the enclosed region bounded by the graphs of the equations $y = x^2$ and $y = -x^2 + 4x + 6$. Begin by finding the points of intersection of the two graphs. From $y = x^2$ and $y = -x^2 + 4x + 6$ we get that

$x^2 = -x^2 + 4x + 6 \quad \longrightarrow$

$2x^2 - 4x - 6 = 0 \quad \longrightarrow$

$x^2 - 2x - 3 = 0 \quad \longrightarrow$

$(x - 3)(x + 1) = 0 \quad \longrightarrow$

$x = 3$ or $x = -1$

Using vertical cross-sections we get that

$$\text{AREA} = \int_{-1}^{3} (top \ - \ bottom) \ dx$$

$$= \int_{-1}^{3} ((-x^2 + 4x + 6) - (x^2)) \ dx$$

$$= \int_{-1}^{3} (-x^2 + 4x + 6) \ dx$$

$$= \Big(-2x^3 3 + 4x^2 2 + 6x\Big)\Big|_{-1}^{3}$$

$$= \Big(-23x^3 + 2x^2 + 6x\Big)\Big|_{-1}^{3}$$

$$= \Big(-23(3)^3 + 2(3)^2 + 6(3)\Big) - \Big(-23(-1)^3 + 2(-1)^2 + 6(-1)\Big)$$

$$= \Big(-18 + 18 + 18\Big) - \Big(23 + 2 - 6\Big)$$

$$= \Big(18\Big) - \Big(23 - 63 - 183\Big)$$

$$= \Big(18\Big) - \Big(-223\Big)$$

$$= 18 + 223$$

$$= 543 + 223$$

$$= 763$$

PROBLEM 5: Compute the area of the enclosed region bounded by the graphs of the equations $y = x^3 + x^2$ and $y = 3x^2 + 3x$.

SOLUTION 2: Compute the area of the enclosed region bounded by the graphs of the equations $y = x^3 + x^2$ and $y = 3x^2 + 3x$. Begin by finding the points of intersection of the two graphs. From $y = x^3 + x^2$ and $y = 3x^2 + 3x$ we get that

$x^3 + x^2 = 3x^2 + 3x \quad \longrightarrow$

$x^3 - 2x^2 - 3x = 0 \quad \longrightarrow$

$x(x^2 - 2x - 3) = 0 \quad \longrightarrow$

$x(x - 3)(x + 1) = 0 \quad \longrightarrow$

$x = 0$, $x = 3$, or $x = -1$

Using vertical cross-sections we get that

$$\text{AREA} = \int_{-1}^{0} (top \ - \ bottom) \ dx + \int_{0}^{3} (top \ - \ bottom) \ dx$$

$$= \int_{-1}^{0} ((x^3 + x^2) - (3x^2 + 3x)) \ dx + \int_{0}^{3} ((3x^2 + 3x) - (x^3 + x^2)) \ dx$$

$$= \int_{-1}^{0} (x^3 - 2x^2 - 3x) \ dx + \int_{0}^{3} (-x^3 + 2x^2 + 3x) \ dx$$

$$= \Big(x^4 4 - 2x^3 3 - 3x^2 2 \Big)\Big|_{-1}^{0} + \Big(- x^4 4 + 2x^3 3 + 3x^2 2 \Big)\Big|_{0}^{3}$$

$$= \Big(0^4 4 - 2(0)^3 3 - 3(0)^2 2 \Big) - \Big((-1)^4 4 - 2(-1)^3 3 - 3(-1)^2 2 \Big) + \Big(- 3^4 4 + 2(3)^3 3 + 3(3)^2 2 \Big)$$

$$- \Big(- 0^4 4 + 2(0)^3 3 + 3(0)^2 2 \Big)$$

$$= \Big(0 \Big) - \Big(14 + 23 - 32 \Big) + \Big(- 814 + 18 + 272 \Big) - \Big(0 \Big)$$

$$= - \Big(312 + 812 - 1812 \Big) + \Big(- 814 + 724 + 544 \Big)$$

$$= - \Big(- 712 \Big) + \Big(454 \Big)$$

$$= 712 + 454$$

$$= 712 + 13512$$

$$= 14212$$

$$= 716$$

PROBLEM 6: Compute the area of the enclosed region bounded by the graphs of the equations $y = \ln x, y = 1$, and $x = e^2$.

SOLUTION 6: Compute the area of the enclosed region bounded by the graphs of the equations $y = \ln x$, $y = 1$ and $y = e^2$. Begin by finding the points of intersection of the two graphs. From $y = \ln x$ and $y = 1$ we get that

$$\ln x = 1 \quad \longrightarrow$$

$$e^{\ln x} = e^1$$

$$x = e$$

Using vertical cross-sections we get that

$$\text{AREA} = \int_e^{e^2} (top \ - \ bottom) \ dx$$

$$= \int_e^{e^2} (\ln x - 1) \ dx$$

$$= \int_e^{e^2} \ln x \ dx - \int_e^{e^2} 1 \ dx$$

Let $A = \displaystyle\int \ln x \ dx$.

Use integration by parts. Let $u = \ln x$ and $dv = dx$ such that $du = 1x \ dx$ and $v = x$.

$$A = x \ln x - \int (1x)(x) \ dx$$

$$= x \ln x - \int \ dx$$

$$= x \ln x - x + C$$

$$A - \int_e^{e^2} 1 \ dx$$

$$= \Big(x \ln x - x - x \Big)\Big|_e^{e^2}$$

$$= \Big(x \ln x - 2x \Big)\Big|_e^{e^2}$$

$$= \Big(e^2 \ln e^2 - 2e^2 \Big) - \Big(e \ln e - 2e \Big)$$

$$= \Big(e^2 (2) - 2e^2 \Big) - \Big(e - 2e \Big)$$

$$= \Big(0 \Big) - \Big(-e \Big)$$

$$= e$$

PROBLEM 7: Compute the area of the enclosed region bounded by the graphs of the equations $y = \cos x, y = \sin x$, and $x = 0$.

SOLUTION 7: Compute the area of the enclosed region bounded by the graphs of the equations $y = \cos x$, $y = \sin x$ and $x = 0$. Begin by finding the points of intersection of the two graphs. From $y = x^2$ and $y = x + 2$ we get that

$\cos x = \sin x \quad \longrightarrow$

$\cos x \sin x = 1 \quad \longrightarrow$

$\cot x = 1 \quad \longrightarrow$

$x = \pi 4$

Using vertical cross-sections we get that

$$\text{AREA} = \int_0^{\pi/4} (top \; - \; bottom) \; dx$$

$$= \int_0^{\pi/4} (\cos x - \sin x) \; dx$$

$$= \Big(\sin x - (-\cos x) \Big) \Big|_0^{\pi/4}$$

$$= \Big(\sin x + \cos x \Big) \Big|_0^{\pi/4}$$

$$= \Big(\sin \pi 4 + \cos \pi 4 \Big) - \Big(\sin 0 + \cos 0 \Big)$$

$$= \Big(\sqrt{2}2 + \sqrt{2}2 \Big) - \Big(0 + 1 \Big)$$

$$= \sqrt{2} - 1$$

PROBLEM 8: Compute the area of the enclosed region bounded by the graphs of the equations $y = 8/x, y = 2x$, and $y = 2$.

SOLUTION 8: Compute the area of the enclosed region bounded by the graphs of the equations $y = 8x$, $y = 2x$ and $y = 2$. Begin by finding the points of intersection of the two graphs. From $y = 8x$ and $y = 2x$ we get that

$8x = 2x \quad \longrightarrow$

$8 = 2x^2 \quad \longrightarrow$

$4 = x^2 \quad \longrightarrow$

$x = 2$ or $x = -2 \quad \longrightarrow$

$y = 4$ or $y = -4$

Using horizontal cross-sections such that $x = 8y$ and $x = y2$ we get that

$$\text{AREA} = \int_2^4 (right \; - \; left) \; dy$$

$$\int_2^4 (8y - y2)\ dy$$

$$= \Big(8 \ln y - y^2 4\Big)\Big|_2^4$$

$$= \Big(8 \ln 4 - 4^2 4\Big) - \Big(8 \ln 2 - 2^2 4\Big)$$

$$= \Big(8 \ln 4 - 4\Big) - \Big(8 \ln 2 - 1\Big)$$

$$= \Big(8 \ln 2^2 - 4\Big) - \Big(8 \ln 2 - 1\Big)$$

$$= \Big(16 \ln 2 - 4\Big) - \Big(8 \ln 2 - 1\Big)$$

$$= 8 \ln 2 - 3$$

PROBLEM 9: Compute the area of the enclosed region bounded by the graphs of the equations $y = \ln x$ and $y = (\ln x)^2$.

SOLUTION 9: Compute the area of the enclosed region bounded by the graphs of the equations $y = \ln x$ and $y = (\ln x)^2$. Begin by finding the points of intersection of the two graphs. From $y = \ln x$ and $y = (\ln x)^2$ we get that

$$\ln x = (\ln x)^2 \quad \longrightarrow$$

$$\ln x - (\ln x)^2 = 0 \quad \longrightarrow$$

$$\ln x(1 - \ln x) \quad \longrightarrow$$

$$\ln x = 0 \text{ or } \ln x = 1 \quad \longrightarrow$$

$$x = 1 \text{ or } x = e$$

Using vertical cross-sections we get that

$$\text{AREA} = \int_1^e (top\ -\ bottom)\ dx$$

$$= \int_1^e (\ln x - (\ln x)^2)\ dx$$

$$= \int_1^e \ln x\ dx - \int_1^e (\ln x)^2\ dx$$

Let $A = \int_1^e \ln x\ dx$ and $B = \int (\ln x)^2\ dx$.

(Recall from *PROBLEM 6* that $\int \ln x\ dx = x \ln -x + C$)

$$A = \Big(x \ln x - x\Big)\Big|_1^e$$

$$= \Big(e \ln e - 1 \ln 1\Big) - \Big(e - 1\Big)$$

$$= \Big(e - 0\Big) - \Big(e - 1\Big)$$

$$= 1$$

Use integration by parts. Let $u = (\ln x)^2$ and $dv = dx$ such that $du = 2 \ln x(1x)\ dx$ and $v = x$.

$$B = \Big(x(\ln x)^2\Big)\Big|_1^e - \int_1^e 2 \ln x\ dx$$

$$= \Big(x(\ln x)^2\Big)\Big|_1^e - 2\Big(x \ln x - x\Big)\Big|_1^e$$

$$= \Big(e(\ln e)^2 - 1(\ln 1)^2\Big) - 2\Big((e \ln e - e) - (1 \ln 1 - 1)\Big)$$

$$= \Big((e - 0)\Big) - 2\Big((e - e) - (0 - 1)\Big)$$

$$= e - 2$$

$$A - B = 1 - (e - 2)$$

$$= 3 - e$$

PROBLEM 10: Compute the area of the enclosed region bounded by the graphs of the equations $y = \tan^2 x$, $y = 0$ and $x = 1$.

SOLUTION 10: Compute the area of the enclosed region bounded by the graphs of the equations $y = \tan^2 x$, $y = 0$, and $x = 1$. Begin by finding the points of intersection of the two graphs. From $y = \tan^2 x$ and $y = 0$ we get that

$$\tan^2 x = 0 \quad \longrightarrow$$

$$\tan x = 0 \quad \longrightarrow$$

$$x = 0$$

Using vertical cross-sections we get that

$$\text{AREA} = \int_0^1 (top\ -\ bottom)\ dx$$

$$= \int_0^1 \tan^2 x\ dx$$

$$= \int_0^1 (\sec^2 x - 1)\ dx$$

$$= \Big(\tan x - x\Big)\Big|_0^1$$

$$= \Big(\tan 1 - 1\Big) - \Big(\tan 0 - 0\Big)$$

$$= \tan 1 - 1$$

PROBLEM 11: Compute the area of the enclosed region bounded by the graphs of the equations $y = x, y = 2x$, and $y = 6 - x$.

SOLUTION 11: Compute the area of the enclosed region bounded by the graphs of the equations $y = x$, $y = 2x$, and $y = 6 - x$. Begin by finding the points of intersection of the two graphs.

From $y = x$ and $y = 2x$ we get that

$x = 2x \quad \longrightarrow$

$-x = 0 \quad \longrightarrow$

$x = 0$

From $y = x$ and $y = 6 - x$ we get that

$x = 6 - x \quad \longrightarrow$

$2x = 6 \quad \longrightarrow$

$x = 3$

From $y = 2x$ and $y = 6 - x$ we get that

$2x = 6 - x \quad \longrightarrow$

$3x = 6 \quad \longrightarrow$

$x = 2$

Using vertical cross-sections we get that

$$\text{AREA} = \int_0^2 (top \ - \ bottom) \ dx + \int_2^3 (top \ - \ bottom) \ dx$$

$$= \int_0^2 (2x - x) \ dx + \int_2^3 ((6 - x) - x) \ dx$$

$$= \int_0^2 x \ dx + \int_2^3 (6 - 2x) \ dx$$

$$= x^2 2 \Big|_0^2 + \Big(6x - 2x^2 2 \Big) \Big|_2^3$$

$$= x^2 2 \Big|_0^2 + \Big(6x - x^2 \Big) \Big|_2^3$$

$$= \Big(2^2 2 - 0^2 2 \Big) + \Big((6 \cdot 3 - 3^2) - (6 \cdot 2 - 2^2) \Big)$$

$$= \Big(2 - 0 \Big) + \Big(9 - 8 \Big)$$

$$= 2 + 1$$

$$= 3$$

PROBLEM 12: Compute the area of the enclosed region bounded by the graphs of the equations

$y = \sin\sqrt{x}$, $y = 0$, and $x = \pi/2$.

SOLUTION 12: Compute the area of the enclosed region bounded by the graphs of the equations $y = \sin\sqrt{x}$, $y = 0$, and $x = \pi/2$. Begin by finding the points of intersection of the two graphs. From $y = \sin\sqrt{x}$ and $y = 0$ we get that

$\sin\sqrt{x} = 0 \quad \longrightarrow$

$x = 0$

Using vertical cross-sections we get that

$$\text{AREA} = \int_0^{\pi/2} (top \ - \ bottom) \ dx$$

$$= \int_0^{\pi/2} (\sin\sqrt{x}) \ dx$$

Use integration by parts. Let $u = \sin\sqrt{x}$ and $dv = dx$ such that $du = \cos\sqrt{x} \cdot 12\sqrt{x} \ dx$ and $v = x$.

$$= x \sin\sqrt{x}\Big|_0^{\pi/2} - \int_0^{\pi/2} \cos\sqrt{x} \cdot 12\sqrt{x} \cdot x \ dx$$

$$= \Big((\pi 2)\sin\sqrt{\pi 2} - (0)\sin\sqrt{0}\Big) - \int_0^{\pi/2} \cos\sqrt{x} \cdot 12\sqrt{x} \cdot x \ dx$$

$$= \pi 2 \sin\sqrt{\pi 2} - \int_0^{\pi/2} \cos\sqrt{x} \cdot 12\sqrt{x} \cdot x \ dx$$

Use substitution. Let

$w = \sqrt{x}$

so that

$dw = 12\sqrt{x} \ dx = \sqrt{x} 2x \ dx$

$x \ dw = \sqrt{x} 2 \ dx \quad \longrightarrow$

$w^2 \ dw = \sqrt{x} 2 \ dx$

$$= \pi 2 \sin\sqrt{\pi 2} - \int_0^{\sqrt{\pi/2}} \cos w \cdot w^2 \ dw$$

Use integration by parts. Let $u' = w^2$ and $dv' = \cos w \ dw$ such that $du' = 2w \ dw$ and $v' = \sin w$.

$$= \pi 2 \sin\sqrt{\pi 2} - \Big(w^2 \sin w\Big|_0^{\sqrt{\pi/2}} - \int_0^{\sqrt{\pi/2}} 2w \sin w \ dw\Big)$$

$$= \pi 2 \sin\sqrt{\pi 2} - w^2 \sin w\Big|_0^{\sqrt{\pi/2}} + \int_0^{\sqrt{\pi/2}} 2w \sin w \ dw$$

$$= \pi 2 \sin\sqrt{\pi 2} - \Big((\sqrt{\pi 2})^2 \sin\sqrt{\pi 2} - (0)^2 \sin 0\Big) + \int_0^{\sqrt{\pi/2}} 2w \sin w \ dw$$

$$= \pi 2 \sin \sqrt{\pi 2} - \pi 2 \sin \sqrt{\pi 2} + \int_0^{\sqrt{\pi/2}} 2w \sin w \ dw$$

$$= \int_0^{\sqrt{\pi/2}} 2w \sin w \ dw$$

Use integration by parts. Let $u" = 2w$ and $dv" = \sin w \ dw$ such that $du" = 2 \ dw$ and $v" = -\cos w$

$$= -2w \cos w \Big|_0^{\sqrt{\pi/2}} + \int_0^{\sqrt{\pi/2}} 2 \cos w \ dw$$

$$= -2\Big((\sqrt{\pi 2}) \cos \sqrt{\pi 2} - (0) \cos 0 \Big) + 2 \sin w \Big|_0^{\sqrt{\pi 2}}$$

$$= -2(\sqrt{\pi 2}) \cos \sqrt{\pi 2} + 2\Big(\sin \sqrt{\pi 2} - \sin 0 \Big)$$

$$= -\sqrt{2\pi} \cos \sqrt{\pi 2} + 2 \sin \sqrt{\pi 2}$$

PROBLEM 13: Compute the area of the enclosed region bounded by the graphs of the equations $x = y^2$ and $x = 4$.

SOLUTION 13: Compute the area of the enclosed region bounded by the graphs of the equations $x = y^2$ and $x = 4$. Begin by finding the points of intersection of the two graphs. From $x = y^2$ and $x = 4$ we get that

$$y^2 = 4 \quad \longrightarrow$$

$$y = -2 \text{ or } y = 2$$

Using horizontal cross-sections we get that

$$\text{AREA} = \int_{-2}^{2} (right \ - \ left) \ dy$$

$$= \int_{-2}^{2} (4 - y^2) \ dy$$

$$= \Big(4y - y^3 3 \Big) \Big|_{-2}^{2}$$

$$= \Big(4 \cdot 2 - 2^3 3 \Big) - \Big(4 \cdot (-2) - (-2)^3 3 \Big)$$

$$= \Big(8 - 83 \Big) - \Big(-8 + 83 \Big)$$

$$= 16 - 163$$

$$= 483 - 163$$

$$= 323$$

PROBLEM 14: Compute the area of the enclosed region bounded by the graphs of the equations $x = y + 3$ and $x = y^2 - y$.

SOLUTION 14: Compute the area of the enclosed region bounded by the graphs of the equations $x = y+3$ and $x = y^2 - y$. Begin by finding the points of intersection of the two graphs. From $x = y+3$ and $x = y^2 - y$ we get that

$y + 3 = y^2 - y \quad \longrightarrow$

$0 = y^2 - 2y - 3 \quad \longrightarrow$

$0 = (y-3)(y+1) \quad \longrightarrow$

$y = 3$ or $y = -1$

Using horizontal cross-sections we get that

$$\text{AREA} = \int_{-1}^{3} (right \ - \ left) \ dy$$

$$= \int_{-1}^{3} ((y+3) - (y^2 - y)) \ dy$$

$$= \int_{-1}^{3} (-y^2 + 2y + 3) \ dy$$

$$= \Big(- y^3 3 + 2y^2 2 + 3y \Big)\Big|_{-1}^{3}$$

$$= \Big(- y^3 3 + y^2 + 3y \Big)\Big|_{-1}^{3}$$

$$= \Big(- 3^3 3 + 3^2 + 3(3) \Big) - \Big(- (-1)^3 3 + (-1)^2 + 3(-1) \Big)$$

$$= \Big(- 9 + 9 + 9 \Big) - \Big(13 + 1 - 3 \Big)$$

$$= \Big(- 9 + 9 + 9 \Big) - \Big(13 + 33 - 93 \Big)$$

$$= \Big(9 \Big) - \Big(- 53 \Big)$$

$$= 273 + 53$$

$$= 323$$

PROBLEM 15: Compute the area of the enclosed region bounded by the graphs of the equations $x = y^3$ and $x = y^2 + 2y$.

SOLUTION 15: Compute the area of the enclosed region bounded by the graphs of the equations $x = y^3$ and $x = y^2 + 2y$. Begin by finding the points of intersection of the two graphs. From $x = y^3$ and $x = y^2 + 2y$ we get that

$y^3 = y^2 + 2y \quad \longrightarrow$

$y^3 - y^2 - 2y = 0 \quad \longrightarrow$

$y(y^2 - y - 2) = 0 \quad \longrightarrow$

$y(y-2)(y+1)=0 \qquad \longrightarrow$

$y=0,\ y=2,\ \text{or}\ y=-1$

Using horizontal cross-sections we get that

$$\text{AREA} = \int_{-1}^{0} (right\ -\ left)\ dy + \int_{0}^{2} (right\ -\ left)\ dy$$

$$= \int_{-1}^{0} (y^3 - (y^2+2y))\ dy + \int_{0}^{2} (y^2+2y-y^3)\ dy$$

$$= \int_{-1}^{0} (y^3 - y^2 - 2y)\ dy + \int_{0}^{2} (y^2+2y-y^3)\ dy$$

$$= \Big(y^4 4 - y^3 3 - 2y^2 2\Big)\Big|_{-1}^{0} + \Big(y^3 3 + 2y^2 2 - y^4 4\Big)\Big|_{0}^{2}$$

$$= \Big(y^4 4 - y^3 3 - y^2\Big)\Big|_{-1}^{0} + \Big(y^3 3 + y^2 - y^4 4\Big)\Big|_{0}^{2}$$

$$= \Big(0^4 4 - 0^3 3 - 0^2\Big) - \Big((-1)^4 4 - (-1)^3 3 - (-1)^2\Big) + \Big(2^3 3 + 2^2 - 2^4 4\Big) - \Big(0^3 3 + 0^2 - 0^4 4\Big)$$

$$= \Big(0\Big) - \Big(14 + 13 - 1\Big) + \Big(83 + 4 - 4\Big) - \Big(0\Big)$$

$$= -\Big(-712\Big) + \Big(83\Big)$$

$$= 712 + 83$$

$$= 712 + 3212$$

$$= 3912$$