Content Creation 2

The following is an example to help in the understanding of Lecture 18

Question: Find the eigenvalues and associated unit eigenvectors of the matrix M

$$M = \begin{pmatrix} 3 & 2 & -5 \\ 2 & -4 & 2 \\ -5 & 2 & 3 \end{pmatrix}$$

First we need to solve the equation det (M-λI) = 0

$$det\begin{pmatrix} 3-\lambda & 2 & -5 \\ 2 & -4-\lambda & 2 \\ -5 & 2 & 3-\lambda \end{pmatrix} = 0$$

$$\Rightarrow \quad (3-\lambda)\big((-4-\lambda)(3-\lambda)-4\big) - 2(2(3-\lambda)+10) - 5\big(4-(-5)(-4-\lambda)\big) = 0$$

$$\Rightarrow \quad -\lambda^3 + 2\lambda^2 + 48\lambda = 0$$

$$\Rightarrow \quad -\lambda(\lambda^2 - 2\lambda - 48) = 0$$

$$\Rightarrow \quad -\lambda(\lambda - 8)(\lambda + 6) = 0$$

The zeroes of det (M-λI) = 0 give the eigenvalues of the matrix.
Therefore λ **= -6, 0, 8** are the eigenvalues of matrix M

Now we need to plug the three eigenvalues back into the matrix

$$\begin{pmatrix} 3-\lambda & 2 & -5 \\ 2 & -4-\lambda & 2 \\ -5 & 2 & 3-\lambda \end{pmatrix}$$

to solve for the eigenvectors. Set the matrix equal to the zero vector and solve for x, y, and z. Note that if your solution has arbitrary values, set the arbitrary variable equal to 1 and solve for the remaining values.

For λ = -6

$$\begin{pmatrix} 9 & 2 & -5 \\ 2 & 2 & 2 \\ -5 & 2 & 9 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Now apply row operations until the matrix is in RREF. Solving for x, y, and z you get

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \mu \\ -2\mu \\ 1\mu \end{pmatrix} = \mu\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

This is your eigenvector for eigenvalue λ = -6

Repeat for λ = 0 which gives

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

This is your eigenvector for eigenvalue λ = 0

Repeat for λ = 8 which gives

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

This is your eigenvector for eigenvalue λ = 8

To find the unit eigenvectors divide each vector by its magnitude.
Your final answer should look like the following

λ = -6, $\begin{pmatrix} \frac{1}{\sqrt{6}} \\ \frac{-2}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \end{pmatrix}$

λ = 0, $\begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix}$

λ = 8, $\begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ \frac{-1}{\sqrt{2}} \end{pmatrix}$

An additional note on multiplicity and dimension:

The multiplicity of each eigenvalue is 1 for this example because each eigenvalue appeared only once from the factors of the characteristic polynomial (the polynomial found by computing det (λI-M) = 0)

The dimension for each eigenspace is 1 for this example because each eigenvalue produced only 1 independent eigenvector