

RREF means that the pivots are equal to one, each pivot is to the right of the previous one, and the rest of the pivot column is zero where pivots are the 1st non-zero entry on each row.

Let's look at the second augmented matrix on problem one of Problems for Gaussian Elimination as an example.

First we need to decide if it is in RREF so let's check the three conditions.

$$\left(\begin{array}{cccccc|c} 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

1. the pivots are all equal to one ✓
2. each pivot is to the right of the previous one ✓
3. the rest of the pivot column is zero ✓

Since all the conditions are met, this matrix is in RREF.

Now let's compute the solution set:

Because some columns don't have pivots, we need to set some variables as arbitrary values, λ

Let's set $x_2 = \lambda_1$, $x_4 = \lambda_2$, and $x_6 = \lambda_3$

now let's solve for the rest of the variables in terms of $\lambda_1, \lambda_2, \lambda_3$

from the augmented matrix,

$$x_1 + x_2 + x_4 + x_6 = 0 \quad \text{plugging in } \lambda_1, \lambda_2, \lambda_3 \text{ and solving for } x_1$$

$$x_1 = -\lambda_1 - \lambda_2 - \lambda_3$$

$$x_3 + 2x_4 + 2x_6 = 0 \quad \text{plugging in } \lambda_2, \lambda_3 \text{ and solving for } x_3$$

$$x_3 = -2\lambda_2 - 2\lambda_3$$

$$x_5 + 3x_6 = 0 \quad \text{plugging in } \lambda_3 \text{ and solving for } x_5$$

$$x_5 = -3\lambda_3$$

Putting this all together you get

$$\left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \lambda_1 \begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \lambda_2 \begin{pmatrix} -1 \\ 0 \\ -2 \\ 1 \\ 0 \\ 0 \end{pmatrix} + \lambda_3 \begin{pmatrix} -1 \\ 0 \\ -2 \\ 0 \\ -3 \\ 1 \end{pmatrix} \right\}$$