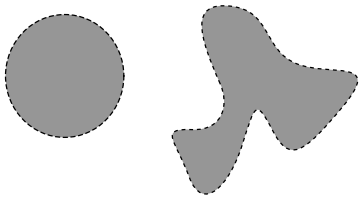
GEOMETRIES OF SURFACES

Reading: Weeks, *The Shape of Space*, Chapters 9, 10, 11

In topology, two shapes that look very different from each other can still be homeomorphic, for example the two below regions are homeomorphic (and the famous donut/coffee cup example).



Although the *topology* of these shapes is the same, the *geometry* of the shapes is very different. The homeomorphism that takes one to the other does not preserve *distances* or *angles*. Even though the topology of a space does not seem to care about the geometry of that space, the reverse works differently. Certain kinds of geometries can only exist on certain kinds of topological spaces. A nice type of geometric space that is commonly studied is one constant curvature. Depending on whether the curvature is positive, zero, or negative, this affects the topological spaces which can possibly have these geometries. A geometry with constant positive curvature is called *spherical*, constant zero curvature is called *flat*, and constant negative curvature is called *hyperbolic*. The goal here is to understand some examples of these different geometries and see what properties distinguish them from each other.

(1) Spherical geometry: Follow the exercises in Chapter 9 of Weeks 9.1-9.6. Calculate more precisely the discussion surrounding figure 9.8–what is the surface area of the hemisphere, what is the radius as measured from the perspective of a flatlander on the sphere (the distance from the north pole to the equator along a great circle), and what would the area of the corresponding disk of that radius be if the disk were flat (compare the two areas)?

(2) Hyperbolic geometry: Follow through Chapter 10 of Weeks. What kinds of phenomena occur in hyperbolic spaces to distinguish them from flat and spherical spaces?

(3) Follow through Weeks Chapter 11. We showed that every surface is homeomorphic to S^2, $T^2\# \cdots \# T^2$, or $\mathbb{P}^2\# \cdots \# \mathbb{P}^2$. Fill out the chart at the end of chapter 11 characterizing which surfaces can have which geometries.