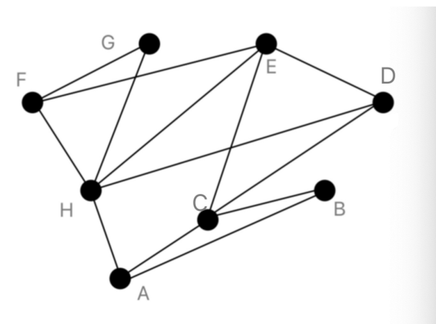
MAT 145: Homework 6 Solution

Xiaochen Liu*

1. One possible answer is $A \to B \to C \to D \to E \to F \to G \to H \to A$.



2. One possible example is K_4.

3. One possible example is K_3.

4. Existence is ensured by connectness of G. Now we prove uniqueness. Assume that v_i and v_j are connected by at least two different paths. For example, $v_i \to a_1 \to \cdots \to a_n \to v_j$ and $v_i \to b_1 \to \cdots \to b_m \to v_j$ are two different paths. Along the path of a's from v_i to v_j, find the first vertex where it diverges from the path of b's, i.e., the smallest k and ℓ such that $a_k = b_\ell$ and $a_{k+1} \neq b_{\ell+1}$. We continue walk along the path of a's, find the first vertex where the two paths meet again, i.e., the smallest $p > k$ such that $a_{p-1} \neq b_{q-1}$ and $a_p = b_q$. Then there are two disjoint paths connecting $a_k(= b_\ell)$ and $a_p(b_q)$. So, there is a cycle in G. A contradiction.

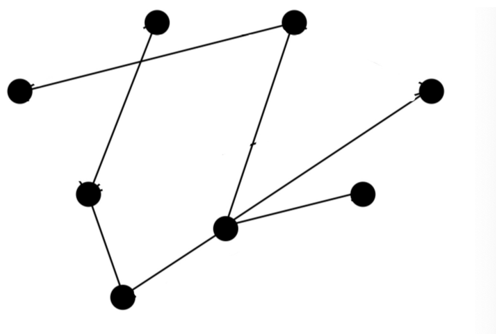
5. Suppose G' is obtained by adding an edge uv to G and $uv \notin E(G)$. In G, u and v are already connected by a path as G is connected. So, there are two disjoint paths connecting u and v, thus there is a cycle.

6. Consider the six 4-vertex trees on page 145. For the leftmost one, for example, call it G_1. Add a new vertex to G_1. This new vertex may connect to any of the four existing vertices. So G_1 has four 'offspring'. Do the same thing for the other five trees. Figures omitted.

*xchliu@math.ucdavis.edu

7. Let e be the number of edges in T, v be the number of vertices in T, v_j be the number of vertices with degree j in T, for $j = 1, 3$. Then we have $e = v - 1$, $e = 2v = v_1 + 3v_3$, $v_1 + v_3 = v$ and $v_3 = 10$. Solving this linear system gives $v_1 = 12$.

8. One possibility is as follow.



9. 0103553.

10. 310442552.