MATH 145 FINAL REFERENCE SHEET

You may use these formulas freely in your solutions.

Binomial Coefficients:

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}$$

Binomial Theorem:

$$(x+y)^n = \sum_{j=0}^{n} \binom{n}{j} x^{n-j} y^j$$

Inclusion-Exclusion:

$$|A \cup B| = |A| + |B| - |A \cap B|$$

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

Pascal's formula:

$$\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$$

Definition: The *degree* of a vertex, $d(v)$ is the number of endpoints of edges which agree with that vertex.

Degree-edge formula:

$$\sum_{v \in V(G)} d(v) = 2\#E(G)$$

Definition: An edge e of a connected graph G is a *cut-edge* if and only if the graph obtained by deleting e from G is not connected.

Definition: A graph is a *tree* if and only if it is connected and contains no cycles.

Theorem: A graph G is a tree if and only if G is connected every edge of G is a cut-edge.

Definition: An *Eulerian walk* is a walk along the edges and vertices of a graph which uses every edge exactly once (can repeat vertices).

Definition: A *Hamiltonian cycle* is a walk along the edges and vertices of a graph which uses every vertex exactly once and then returns to the original vertex (it does not need to use all the edges, but it cannot use an edge more than once).

Theorem: A graph has a closed Eulerian walk if and only if all the vertices of the graph have even degree. A graph has a non-closed Eulerian walk if and only if exactly two vertices of the graph have odd degree.

Theorem: Kruskal's greedy algorithm produces the lowest cost spanning tree.

Euler's formula: If G is a connected graph with a planar embedding, the relationship between the number of vertices, edges, and regions is:

$$\#V(G) - \#E(G) + \#R(G) = 2$$