Practice Final Exam 1

Math 145, Spring 2019

Name: ____

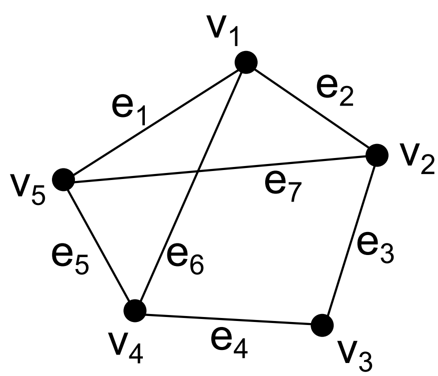
Student ID: __________________________________

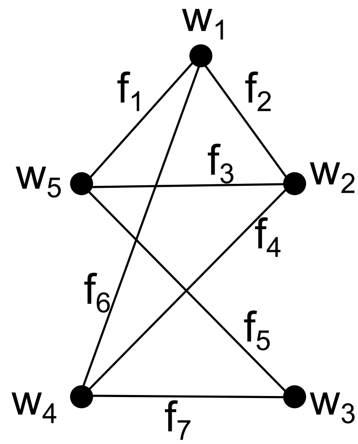
Every solution must contain an explanation written in words supporting your numerical solution to receive credit.

You do not need to simplify numerical expressions for your final answers (e.g. you can write $2^8 \cdot 4!$ instead of multiplying out to 6144.)

If you need extra space for your solutions, there is an extra page at the back of the exam. If you need extra space for any problem, write CONTINUED IN EXTRA SPACE on the page where the problem is given to you. In the extra space write the problem number that you are solving in that space.

Problem 1: For the following pair of graphs, either prove they are isomorphic or prove they are not isomorphic.





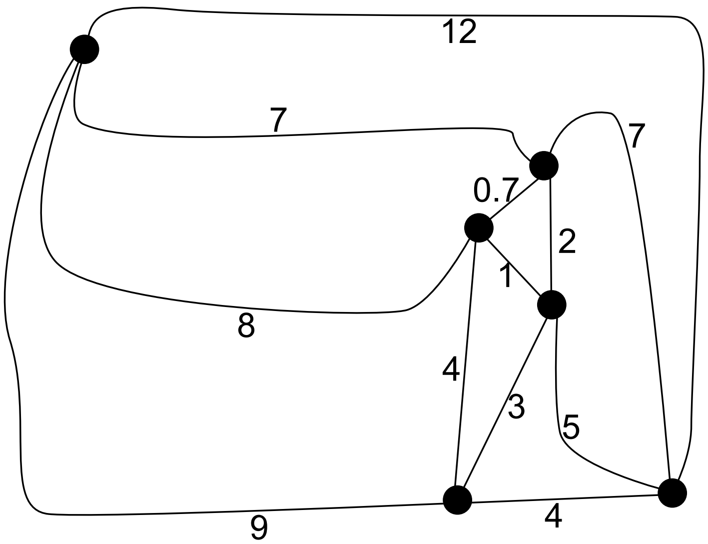
Problem 2: At a store opening, they were giving out a free pencil to every 5^{th} customer who walked in (the 5^{th}, 10^{th}, 15^{th}, etc.), a free hat to every 12^{th} customer who walked in (the $12^{th}, 24^{th}, \ldots$), and a free T-shirt to every 23^{rd} customer who walked in $(23^{rd}, 46^{th}, \ldots)$. If there were 500 customers, how many got a free item?

Problem 3: Give the proof for the statement (directly from the definitions) that if G is a graph, and $d(v)$ denotes the degree of a vertex then

$$\sum_{v \in V(G)} d(v) = 2\#E(G).$$

Problem 4: Suppose G is a connected graph, and e an edge of G. Prove that e is NOT a cut-edge *if and only if* it is contained in a cycle of G.

Problem 5: Find the minimal cost spanning tree for the following weighted graph. Draw the spanning tree and determine its total cost.



Problem 6: Suppose G is a simple graph with 10 vertices and 28 edges. Prove that there are at least two vertices v_1 and v_2 such that $d(v_1) + d(v_2) \geq 12$. Next prove there are at least two other vertices v_3 and v_4 such that there is are edges connecting v_1 to v_3, v_1 to v_4, v_2 to v_3 and v_2 to v_4. Conclude G contains a cycle of length 4.

Problem 7: Suppose G is a graph and v_1 and v_2 are two vertices in G. Suppose there exists a walk starting at v_1 and ending at v_2. Prove that G has a linear subgraph where v_1 and v_2 are the endpoints.

Problem 8: Sam is scheduling a family vacation. The trip could last 5, 6 or 7 days. Based on the other things the family has scheduled during the summer, the trip must lie within the dates July 1 through July 22 (including July 1 and 22). How many different possibilities are there for the length and dates of the trip?

Problem 9: Suppose G is a connected planar graph, and there is a planar polygonal embedding such that every region has at least 5 edges in its sides. If G has 15 edges. What is the smallest number for the number of vertices of G? Give an example of a planar graph with this number of vertices and edges with the criterion that every region has at least 5 edges in its sides.

Extra Space: