Homework 5

Math 145, Spring 2019

Every solution must contain an explanation written in words supporting your numerical solution to receive credit.

1. A bipartite graph $K_{m,n}$ is a simple graph with vertex set $V(K_{m,n}) = \{v_1, \cdots, v_m, w_1, \cdots, w_n\}$ such that there is an edge between every pair v_i, w_j, but there are no edges between two v_i's and no edges between two w_j's.

   (a) What are the values of $\#V(K_{m,n})$ and $\#E(K_{m,n})$?

   (b) What is the degree sequence for $K_{m,n}$ (the set of degrees for each of the vertices)?

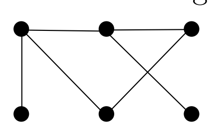
   Definition: The complete graph K_n is a simple graph with n vertices where there is an edge connecting every pair of vertices.

2. Determine the degree sequences for the following graphs:

   (a) A linear graph with n vertices.

   (b) K_n

   (c) The 6 vertex graph shown here:

3. Let G be any *simple* graph with $\#V(G) = n$. Prove that G is isomorphic to a subgraph of K_n.

4. For each of the following sequences, either draw a *simple* graph which has that as a its degree sequence, or prove that there is no simple graph with that degree sequence.

   (a) $(7, 2, 2, 2)$

   (b) $(3, 3, 2, 2)$

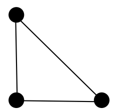
   (c) $(2, 2, 2, \cdots, 2)$ (n vertices each with degree 2 with $n \geq 3$)

   (d) $(5, 3, 2, 2, 2, 1)$

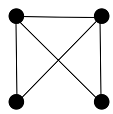
5. Let G_1 and G_2 be simple graphs which are *isomorphic*. Let $(d_1, d_2, \cdots, d_n)$ be the degree sequence of G_1. Prove that the degree sequence of G_2 is a reordering of $(d_1, d_2, \cdots, d_n)$.

6. Prove that the following graphs are connected:

   (a) The 3 vertex cycle:

   (b) The following 4 vertex graph:

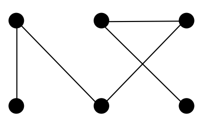
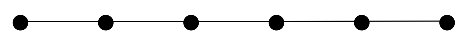
   

   (c) K_n

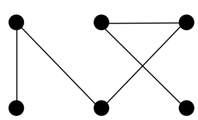
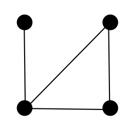
7. An edge e of a connected graph G is called a *cut edge* if the graph G' obtained by deleting that edge ($V(G') = V(G)$ and $E(G') = E(G) \setminus \{e\}$) is not connected. Prove that if G_1 and G_2 are connected simple graphs which are isomorphic and if G_1 has a cut edge, then G_2 also has a cut edge.

8. For the following pairs of graphs G_1 and G_2, either prove they are isomorphic by constructing the isomorphism, or prove they are not isomorphic.

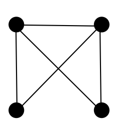
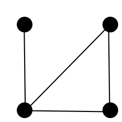
   (a) G_1 and G_2 below (choose your labels for the vertices and edges)

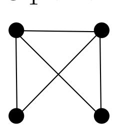
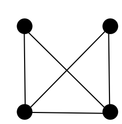
   (b) G_1 and G_2 below (choose your labels for the vertices and edges)

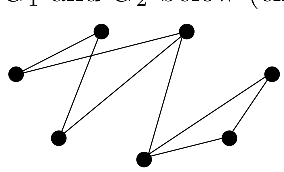
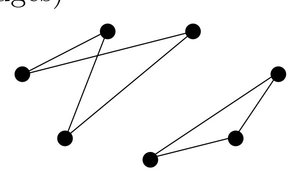
   (c) G_1 and G_2 below (choose your labels for the vertices and edges)

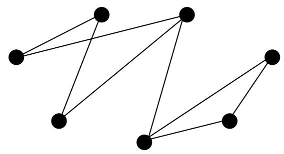
   (d) G_1 and G_2 below (choose your labels for the vertices and edges)

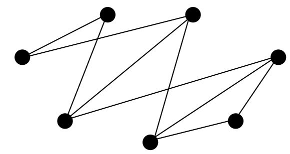
    

   (e) G_1 and G_2 below (choose your labels for the vertices and edges)

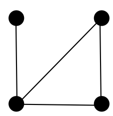
(f) G_1 and G_2 below (choose your labels for the vertices and edges)



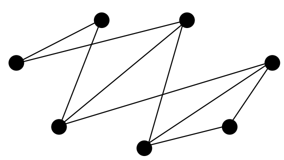


9. For the following graphs, determine whether or not it has an Eulerian walk. If it has an Eulerian walk, find one (indicate your starting point and ending point and trace out the path of the walk next to the graph). If it does not have an Eulerian walk, prove it.

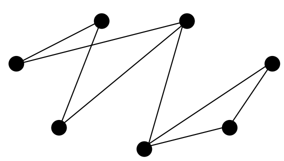
   (a) G is

   (b) G is

   (c) G is

   (d) G is K_5

   (e) G is K_6