Extra Practice for Midterm 2

Math 147, Fall 2018

Name:

Problem 1: Let $(\mathbb{R}, \tau_{Euc})$ be the real line with the Euclidean topology.

(a) Give an example of a subspace A of $\mathbb{R}$ where A has the subspace topology τ_A, and a subset $U \subset A$ such that the interior of U as a subset of A is the *same* as the interior of U as a subset of $\mathbb{R}$.

(b) Give an example of a subspace B of $\mathbb{R}$ where A has the subspace topology τ_B, and a subset $V \subset B$ such that the interior of B as a subset of B is *different* than the interior of V as a subset of $\mathbb{R}$.

Problem 2: Let (X, τ_X) and (Y, τ_Y) be regular spaces. Prove that the product $(X \times Y, \tau_X \times \tau_Y)$ is regular. [Hint: use the equivalent definition of regularity, that a space is regular if and only if for every point x and open set U containing x, there is an open subset V such that $x \in V$ and $\overline{V} \subset U$.]

Problem 3: Let X be the set of natural numbers $1, 2, 3, \cdots$. We define a topology τ' on X as follows. $U \subset X$ is open in τ' if and only if either one or both of the following two conditions hold

1. $X \setminus U$ is finite
2. $1 \in X \setminus U$

Prove that X with the topology τ' is *not connected*.

Problem 4: Let $X = \mathbb{R}^2$ with the Euclidean topology. Define an equivalence relation $\sim$ on X by $(x_1, y_1) \sim (x_2, y_2)$ if and only if $x_1 + 2y_1^2 = x_2 + 2y_2^2$. Let $Y = \mathbb{R}$ with the Euclidean topology. Construct a map $f : X/_\sim \to Y$ and show that f is *well-defined, continuous and has an inverse*. You do NOT need to prove that f^{-1} is continuous (f^{-1} probably will be continuous, you just do not need to prove it).