Extra Practice for Midterm 1

Math 147, Fall 2018

Problem 1: Suppose $f : X \to Y$ is a function between sets X and Y. Suppose $\mathcal{B}_Y$ is a basis for a topology on Y.

(a) Show that if we define the collection

$$\mathcal{B}_X = \{f^{-1}(B) \mid B \in \mathcal{B}_Y\}$$

then $\mathcal{B}_X$ is a basis on X.

(b) If we give X the topology τ_X generated by the basis $\mathcal{B}_X$ and Y the topology generated by the basis $\mathcal{B}_Y$, show that $f : (X, \tau_X) \to (Y, \tau_Y)$ is a continuous function.

Problem 2: Let d_X be a metric on a set X and d_Y be a metric on a set Y.

(a) Show that the function $d : (X \times Y) \times (X \times Y) \to \mathbb{R}$ defined by

$$d((x_1, y_1), (x_2, y_2)) = \max\{d_X(x_1, x_2), d_Y(y_1, y_2)\}$$

is a metric on $X \times Y$.

(b) Show that in this metric $B_\varepsilon((x, y)) = B_\varepsilon(x) \times B_\varepsilon(y)$.

Problem 3: Let $f : (\mathbb{R}, \tau_1) \to (\mathbb{R}, \tau_2)$ be the function $f(x) = 2x$.

(a) If τ_1 is the discrete topology and τ_2 is the Euclidean topology, determine whether or not f is continuous and prove it.

(b) If τ_1 is the Euclidean topology and τ_2 is the discrete topology, determine whether or not f is continuous and prove it.

(c) If τ_1 is the trivial topology defined by $\tau_1 = \{\emptyset, \mathbb{R}\}$ and τ_2 is the Euclidean topology, determine whether or not f is continuous and prove it.

Problem 4: Let $A \subset X$ be a subset of a topological space. Show that $Bdry(A) = \emptyset$ if and only if A is both open and closed in X.