Practice Midterm 2

Math 147, Fall 2018

Name:

Problem 1: Let (X, τ_X) be a topological space. Let Y be a subspace of X with the subspace topology τ_Y. Let $A \subseteq Y$. Let $\overline{A}^X$ denote the closure of A in (X, τ_X) and let $\overline{A}^Y$ denote the closure of A in (Y, τ_Y).

(a) Prove that $\overline{A}^Y \subset \overline{A}^X$.

(b) Give an example where $\overline{A}^Y \neq \overline{A}^X$.

Problem 2: Let $X = \mathbb{R} \setminus \{0\} = (-\infty, 0) \cup (0, \infty)$ with the Euclidean topology τ. Define an equivalence relation $\sim$ on X by $x_1 \sim x_2$ if and only if $x_1 = cx_2$ where $c > 0$. Let Y be the set with two elements $Y = \{+, -\}$. Let τ_Y be the discrete topology on Y. Construct a map $f : X/_\sim \to Y$ and show that f is *well-defined, continuous and has an inverse.* You do NOT need to prove that f^{-1} is continuous (f^{-1} probably will be continuous, you just do not need to prove it).

Problem 3: If X is a metric space, and τ is the topology induced by the metric (with basis the open balls of positive radius), show that (X, τ) is Hausdorff.

Problem 4: Let A and B be subsets of a topological space (X, τ) such that (A, τ_A) and (B, τ_B) are connected with the subspace topology. If $A \cap B \neq \emptyset$ show that $A \cup B$ with the subspace topology is connected.