Homework 5

Math 147, Fall 2018

1. Consider the subset $Y = [-1, 1]$ of the real line $\mathbb{R}$. Using the usual topology on $\mathbb{R}$, give Y the subspace topology. Which of the following subsets are open sets in Y with the subspace topology? Which are open sets in $\mathbb{R}$?

$$A = \{x \mid \frac{1}{2} < |x| < 1\}$$

$$B = \{x \mid \frac{1}{2} < |x| \leq 1\}$$

$$C = \{x \mid \frac{1}{2} \leq |x| < 1\}$$

$$D = \{x \mid \frac{1}{2} \leq |x| \leq 1\}$$

$$E = \{x \mid 0 < |x| < 1 \text{ and } 1/x \notin \mathbb{Z}_+\}$$

2. Let Y be a subspace of X and let A be a subset of Y. Let $\overline{A}^X$ denote the closure of A in X and let $\overline{A}^Y$ denote the closure of A in Y. Prove that $\overline{A}^Y \subset \overline{A}^X$. Give an example where $\overline{A}^Y \neq \overline{A}^X$.

3. Let $X = \Pi_{\alpha \in I} X_\alpha$ be a topological product of the family of spaces $\{X_\alpha\}_{\alpha \in I}$. Let $p_\alpha : X \to X_\alpha$ denote the projection map. Prove that a function $f : Y \to X$ from a space Y to X is continuous if and only if for each $\alpha \in I$, $p_\alpha \circ f : Y \to X_\alpha$ is continuous.

4. Consider the product $\Pi_{i=1}^{\infty} \mathbb{R}_i$ where $\mathbb{R}_i = \mathbb{R}$ of infinitely many copies of $\mathbb{R}$. Its open sets using the *product topology* have the form $U_1 \times \cdots \times U_n \times \mathbb{R} \times \mathbb{R} \times \cdots$ for open sets $U_i \subset \mathbb{R}$. Consider the subset $C \subset \mathbb{R}^\infty$ consisting of sequences $(c_1, c_2, \cdots)$ such that $c_i \neq 0$ for only finitely many values of i.
   (a) What is the closure of C, $\overline{C}$ in $\mathbb{R}^\infty$ with the product topology?
   (b) Another topology on a product such as $\mathbb{R}^\infty$ is called the *box topology*. The open sets are generated by the basis of sets of the form $\Pi_{i=1}^{\infty} U_i$ where $U_i \subset \mathbb{R}$ is open for every $i = 1, 2, \cdots$. What is the closure of C in $\mathbb{R}^\infty$ with the box topology?