Practice Final Exam

Math 147, Fall 2018

Name:

Problem 1: Consider the topology τ_N on $\mathbb{R}$ given by

$$\tau_N = \{(-x, x) | x > 0\} \cup \{\emptyset, \mathbb{R}\}$$

(a) Show that τ_N does give a topology (satisfies the three axioms defining a topology).

(b) Show that $(\mathbb{R}, \tau_N)$ is not Hausdorff

(c) Show that $(\mathbb{R}, \tau_N)$ is connected.

(d) What is the closure of the set $(3, 4)$ in $(\mathbb{R}, \tau_N)$?

(e) Show that $(\mathbb{R}, \tau_N)$ is not compact.

(f) Let τ_{Euc} be the Euclidean topology on $\mathbb{R}$ and let $f : (\mathbb{R}, \tau_N) \to (\mathbb{R}, \tau_{Euc})$ be the identity function defined by $f(x) = x$. Show that f is *NOT* continuous.

Problem 2: Let (X, d) be a metric space.

(a) For each $x \in X$ and $n \in \mathbb{N}$ a positive integer, let $B_{1/n}(x) = \{y \in X \mid d(x, y) < 1/n\}$. Let $\mathcal{B} = \{B_{1/n}(x)\}$ indexed over all $x \in X$ and $n \in \mathbb{N}$. Show that $\mathcal{B}$ is a basis.

(b) Let τ be the topology on X induced by the metric d. Let $x_0 \in X$ be any fixed point. Let $C = \{y \in X \mid d(y, x_0) = 5\}$. Show that C is closed.

Problem 3: Suppose (X, τ) is a topological space which is *compact*. Prove that if $C \subset X$ is a *closed subset*, then C is compact.

Problem 4: Let $X = \{(x, y) \in \mathbb{R}^2 \mid 1 \leq x^2 + y^2 \leq 4\}$ with the Euclidean topology. Define an equivalence relation on X by $(x, y) \sim (x', y')$ if and only if $x' = cx$ and $y' = cy$ for some $c > 0$. Let $S^1 = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1\}$. Prove that the quotient $X/\sim$ is homeomorphic to S^1 by defining a map $f : X/\sim \to S^1$ which is well-defined, continuous, and has a continuous inverse.

Problem 5: Let (X, τ_X) and (Y, τ_Y) be compact topological spaces and let $(X \times Y, \tau_{X\times Y})$ be the product space with the product topology. Let $y_0 \in Y$ be a point. Consider the subspace $X \times \{y_0\} \subset X \times Y$ with the subspace topology τ_{sub}. ($X \times \{y_0\} = \{(x, y_0) \mid x \in X\}$). Prove that (X, τ_X) is homeomorphic to $(X \times \{y_0\}, \tau_{sub})$ by defining a map $f : (X, \tau_X) \to (X \times \{y_0\}, \tau_{sub})$ and showing that it is continuous and has continuous inverse.

Problem 6: Show that $f : (X, \tau_X) \to (Y, \tau_Y)$ is a continuous function if and only if for every *closed* subset of Y, $C \subset Y$, the preimage $f^{-1}(C)$ is closed in X.