Practice Midterm 1

Math 147, Fall 2018

Problem 1:

(a) Let (X, τ) be a topological space. Suppose V_α is closed in (X, τ) for each $\alpha \in \mathcal{I}$. Prove that the intersection $\cap_\alpha V_\alpha$ is closed.

(b) Consider $\mathbb{R}$ with the Euclidean topology. Give an example of a infinite collection of closed subsets V_i such that the infinite union $V = \cup_i V_i$ is not closed. (Prove V is not closed in your example.)

Problem 2: Let S^1 be the unit circle in $\mathbb{R}^2$ with the Euclidean metric topology given by

$$S^1 = \{(x_1, x_2) \mid d_{Euc}((x_1, x_2), (0, 0)) = 1\}.$$

Show that S^1 is a closed subset of $\mathbb{R}^2$.

Problem 3: We define a topology on $\mathbb{R}$ by $\tau_N = \{(-x, x) \mid x > 0\} \cup \{\emptyset, \mathbb{R}\}$.

(a) Prove that τ_N is a topology (satisfies the 3 axioms).

(b) What is the closure of the subset $(5, 6)$ in the topology τ_N?

Problem 4: Let $\mathcal{B}$ be the collection of open parallelograms in $\mathbb{R}^2$ of the form $B_{a,b,c,d} = \{(x,y) \in \mathbb{R}^2 \mid y+a < x < y+b,\ c < y < d\}$ where $a,b,c,d \in \mathbb{R}$ are constants.

(a) Prove that $\mathcal{B}$ is a basis on $\mathbb{R}^2$ (satisfies the two axioms).

(b) Prove that the unit square $\{(x,y) \mid 0 < x < 1,\ 0 < y < 1\}$ is open in the topology generated by $\mathcal{B}$.