Practice Midterm 2

Math 147, Fall 2018

Name:

Problem 1: Consider the subset $Y = [-1, 1]$ of the real line $\mathbb{R}$. Let $\mathbb{R}$ have the Euclidean topology τ_{Euc} and let τ_Y denote the subspace topology on Y.

(a) Is $A = \{x \mid \frac{1}{2} < |x| \leq 1\}$ open in τ_Y? Prove it is or is not.

(b) Is $B = \{x \mid \frac{1}{2} \leq |x| < 1\}$ open in τ_Y? Prove it is or is not.

Problem 2: Let $X = \mathbb{R}^2$ with the Euclidean topology. Define an equivalence relation $\sim$ on X by $(x_1, x_2) \sim (z_1, z_2)$ if and only if $x_1^2 + x_2^2 = z_1^2 + z_2^2$. Let $Y = [0, \infty)$ with the Euclidean topology. Construct a map $f : X/_\sim \to Y$ and show that f is *well-defined, continuous and has an inverse.* You do NOT need to prove that f^{-1} is continuous (f^{-1} probably will be continuous, you just do not need to prove it).

Problem 3: Let τ_{Zar} be the Zariski topology on $\mathbb{R}$. Remember that a set $U \subset \mathbb{R}$ is open in the Zariski topology if and only if $\mathbb{R} \setminus U$ is finite or $U = \emptyset$. Prove that $(\mathbb{R}, \tau_{Zar})$ is not Hausdorff.

Problem 4: Suppose $f : (X, \tau) \to (\{1, 2, 3\}, \tau_{dis})$ is a continuous *surjective* function (τ_{dis} is the discrete topology). Prove that there are non-empty closed subsets, A, B, and C of X such that $X = A \cup B \cup C$ and $A \cap B = A \cap C = B \cap C = \emptyset$.