HOMEWORK 2 (MAT 215A) TOPOLOGY

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(1) Prove that the suspension over any nonempty path connected space is simply connected.

(2) Prove that the join of two nonempty path connected spaces is simply connected.

(3) Use Seifert van Kampen to prove that $\pi_1(S^n)$ is trivial if $n > 1$.

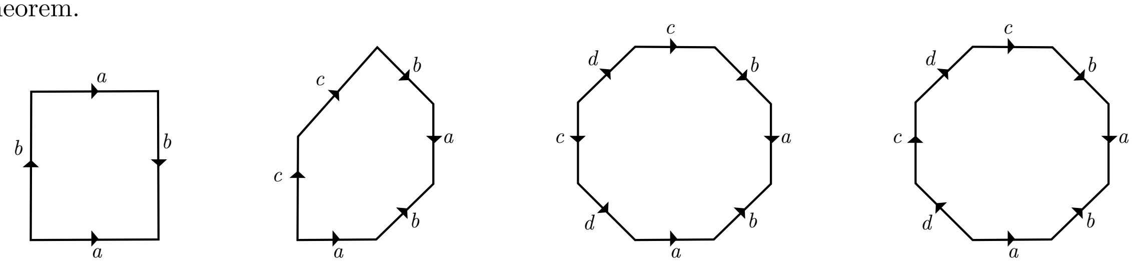
(4) We know the torus with a disk removed $T^2 \backslash D^2$ is homotopy equivalent to $S^1 \vee S^1$ so its fundamental group is the free group on two generators. If γ is a circle parallel to the boundary of the disk which was removed and we let x_0 be a point on that circle, what element of $\pi_1(T^2 \backslash D^2, x_0)$ does that circle represent?

(5) The *connected sum* of two surfaces Σ_1 and Σ_2 is obtained by cutting out a disk from each surface and gluing the resulting surfaces along their circle boundaries:

$$\Sigma_1 \# \Sigma_2 = (\Sigma_1 \backslash D_1) \cup_{S^1} (\Sigma_2 \backslash D^2)$$

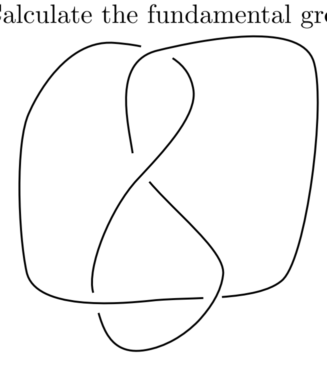
Using the natural decomposition into two pieces given by the connected sum and the Seifert van Kampen theorem, calculate $\pi_1(T^2 \# T^2)$, $\pi_1(\mathbb{RP}^2 \# T^2)$, and $\pi_1(\mathbb{RP}^2 \# \mathbb{RP}^2)$.

(6) Just as the torus can be represented by identifying opposite sides of a square, other surfaces can be represented by identifying certain sides of various polygons. Calculate the fundamental groups of the surfaces resulting from the following polygonal identifications using the Seifert-van Kampen theorem.



Can you identify the fundamental groups of these surfaces with fundamental groups of the connected sum surfaces from the previous problem?

(7) Calculate the fundamental group of the complement of the figure eight knot shown below:



(8) Calculate the fundamental group of the complement of the Borromean rings shown below: