Lecture 18

Friday, February 19, 2021 2:06 PM

<u>HW5 is posted</u>

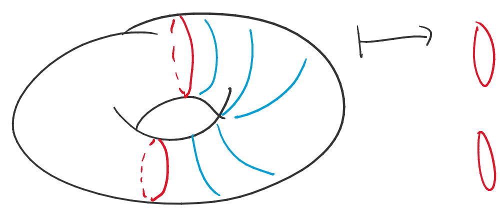
$f: (X,A) \to (Y,B)$

$$\begin{tikzcd} \cdots \arrow[r] & H_n(A) \arrow[r, "i_*"] \arrow[d, "f_*"] & H_n(X) \arrow[r, "j_*"] \arrow[d, "f_*"] & H_n(X,A) \arrow[r, "\partial"] \arrow[d] & H_{n-1}(A) \arrow[r] \arrow[d] & \cdots \\ & H_n(B) \arrow[r] & H_n(Y) \arrow[r] & H_n(Y,B) \arrow[r] & H_{n-1}(B) & \end{tikzcd}$$

$$\begin{tikzcd} 0 \arrow[r] & A \arrow[r, "i"] \arrow[d, "f"] & B \arrow[r, "j"] \arrow[d, "g"] & C \arrow[r] \arrow[d, "h"] & 0 \\ 0 \arrow[r] & A' \arrow[r, "i'"'] & B' \arrow[r, "j'"'] & C' \arrow[r] & 0 \end{tikzcd}$$

$$\begin{tikzcd} & & & A_{n-1} \arrow[d] & a \arrow[d] \\ b \arrow[d, mapsto] & B_n \arrow[r, "d_n"] \arrow[d] & & B_{n-1} & db \\ c \in & C_n & & & \end{tikzcd}$$

$H_2(X,A) \xrightarrow{\partial} H_1(A)$



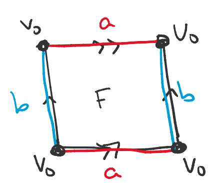
$$\begin{tikzcd} F \arrow[r] & E \arrow[d, "p"] \\ & B \end{tikzcd}$$

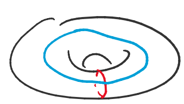
p fibration

$p^{-1}(b) \cong F$

$$\cdots \to \pi_n(F) \to \pi_n(E) \to \pi_n(B) \to \pi_{n-1}(F) \to \cdots$$

<u>Example:</u> T^2





<u>Cells:</u> 0-cell : v_0

2 1-cells : a b

1 2-cell : F

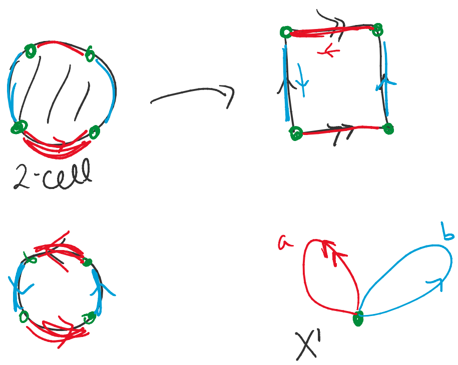
$$\cdots \to 0 \to \underset{C_2^{cw}}{\mathbb{Z}\langle F\rangle} \xrightarrow{d_2^{cw}} \underset{C_1^{cw}}{\mathbb{Z}\langle a,b\rangle} \xrightarrow{d_1^{cw}} \underset{C_0^{cw}}{\mathbb{Z}\langle v_0\rangle} \xrightarrow{d_0^{cw}} 0$$

$d_1(a) = v_0 - v_0 = 0$

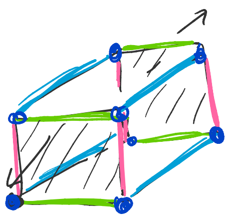
2 1-cells : a b
1 2-cell : F

$d_1(a) = V_0 - V_0 = 0$
$d_1(b) = V_0 - V_0 = 0$

$d_2(F) = \underbrace{\deg_a(\text{attaching map})}_{\underbrace{1-1}_{0}} a + \underbrace{\deg_b(\text{attaching map})}_{\underbrace{1-1}_{0}} b = 0$



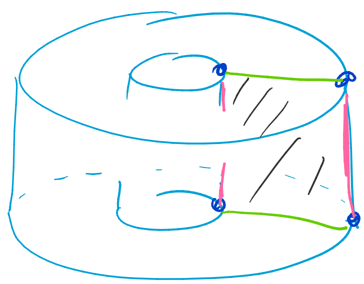
T^3 :
$S^1 \times S^1 \times S^1$



identify opposite faces

<u>cell decomp</u>:
1 0-cell
3 1-cells
3 2-cells
1 3-cell

$$0 \to \mathbb{Z} \xrightarrow{d_3 = 0} \mathbb{Z}^3 \xrightarrow{d_2 = 0} \mathbb{Z}^3 \xrightarrow{d_1 = 0} \mathbb{Z} \xrightarrow{d_0} 0$$



Morse theory

<u>Note</u>: For any n-diml closed <u>orientable</u> manifold, there is a cell decomposition with a single n-cell (no higher dimal cells) such that $\underline{d_n^{CW}}(n\text{-cell}) = 0$

$\Rightarrow H_n(M) \cong \mathbb{Z}$

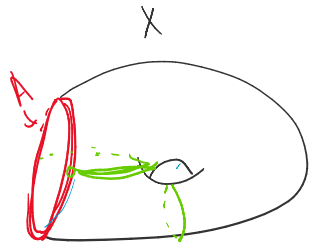
Every n-1 cell in the (n-1) skeleton is 2-sided

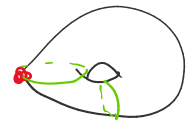
Look at $\mathbb{RP}^2$ for counterexample in non orientable case

$$0 \to \overset{C_2}{\mathbb{Z}} \xrightarrow{\times 2} \mathbb{Z} \xrightarrow{0} \mathbb{Z} \to 0$$

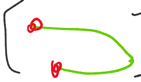
$H_2(\mathbb{RP}^2) = 0$

$H_1(\mathbb{RP}^2) = \mathbb{Z}/2\mathbb{Z}$



$X/A \simeq$ 

$H_2(X,A)$  represents an elt of $H_2(X,A)$

$\Big[$  $\Big]$ represents an elt of $H_1(X,A)$

$=$

$\Big[$  $\Big]$ in $H_1(X,A)$