HOMEWORK 6 (MAT 215B) TOPOLOGY

LAURA STARKSTON

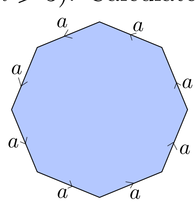
(1) Use the Mayer-Vietoris sequence to show that for a space X, the suspension ΣX (defined as $X \times [0,1]/\sim$ where $(x,0) \sim (x',0)$ and $(x,1) \sim (x',1)$ for all $x, x' \in X$) satisfies $\tilde{H}_n(\Sigma X) \cong \tilde{H}_{n-1}(X)$ for all n.

(2) Use the Mayer-Vietoris sequence and your prior homology calculations from previous homeworks to calculate homology for the following spaces

   (a) The space obtained by gluing a Mobius band along its boundary to the circle $\mathbb{R}\mathrm{P}^1$ inside of $\mathbb{R}\mathrm{P}^2$.

   (b) The space obtained by gluing F, a torus with a disk deleted, along its boundary, to the circle m inside of a closed torus T, where m is a meridian (one of the generators of $H_1(T)$).

(3) Recall the space X from Homework 4, the space obtained from identifying all n sides of an n-gon in a counter-clockwise orientation (assume $n \geqslant 3$). Calculate $H_k(X;\mathbb{Z}_n)$, $H_k(X;\mathbb{Q})$, and $H_k(X;\mathbb{Z}_2)$.



(4) Let H_1 and H_2 be two copies of the 3-dimensional handlebody of genus g. This means they are "filled in genus g surfaces" or more formally obtained by gluing g 3-dimensional 1-handles ($I \times D^2$) to a 3-ball along embeddings of $\{0,1\} \times D^2$. Note that 3-dimensional handlebodies of genus g deformation retract to a wedge of g circles. The boundary of each handlebody is a genus g surface. Let X be the space obtained by gluing the boundary of H_1 to the boundary of H_2 via the identity map. Use the Mayer-Vietoris sequence to calculate the homology groups of X.

   If you want a challenge, you can see if you can calculate homology when you glue the two handlebodies by a different map (instead of the identity). You can explore for different values of g and different homeomorphisms, what you get. When a 3-manifold is presented as the gluing of two handlebodies in this way, it is called a *Heegaard decomposition.*