Solution to odd number problem.

(1). $S = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \middle| \; 3x_1 + x_3 = 0 \right\}$. Let. $\begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix}, \begin{bmatrix} a_2 \\ b_2 \\ c_2 \end{bmatrix} \in S$, $r \in \mathbb{R}$.

$$\begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix} + \begin{bmatrix} a_2 \\ b_2 \\ c_2 \end{bmatrix} = \begin{bmatrix} a_1+a_2 \\ b_1+b_2 \\ c_1+c_2 \end{bmatrix}. \quad 3(a_1+a_2) + c_1+c_2 = (3a_1+c_1) + (3a_2+c_2) = 0$$

$$r\begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix} = \begin{bmatrix} ra_1 \\ rb_1 \\ rc_1 \end{bmatrix} \quad 3(ra_1) + rc_1 = r(3a_1+c_1) = 0$$

(3). $S = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \middle| \; x_1 + 2x_2 + x_3 = 5 \right\}$, $\begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix} \in S$.,

But. $\begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 5 \end{bmatrix}$, $5 + 2\cdot 0 + 5 = 10 \neq 5$.

(5) $$A^T = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 3 \\ -1 & -2 & 1 \\ 2 & 3 & -5 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \\ 0 & -1 & -3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$ $\Rightarrow$ 2 dimension hence a plane.

(7). $$\begin{bmatrix} 0 & 0 & 0 \\ -1 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix} \text{ OR } \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$ $\to$ null space is everything.

(9). $$A = \begin{bmatrix} 0 & -1 & 2 & 3 \\ 1 & -3 & -1 & 2 \\ 1 & -4 & 1 & 5 \end{bmatrix} \to \begin{bmatrix} 1 & -3 & -1 & 2 \\ 1 & -4 & 1 & 5 \\ 0 & -1 & 2 & 3 \end{bmatrix} \to \begin{bmatrix} 1 & -3 & -1 & 2 \\ 0 & -1 & +2 & 3 \\ 0 & -1 & 2 & 3 \end{bmatrix} \to \begin{bmatrix} 1 & -3 & -1 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\to \begin{bmatrix} 1 & 0 & -7 & -7 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \to \begin{matrix} x_1 = 7x_3 + 7x_4 \\ x_2 = 2x_3 + 3x_4 \end{matrix} \Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = x_3 \begin{bmatrix} 7 \\ 2 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 7 \\ 3 \\ 0 \\ 1 \end{bmatrix}$$

(11) $\left[\begin{array}{cccc|c} 1 & -2 & 0 & -1 & 2 \\ 1 & -2 & 0 & 0 & 5 \\ 1 & -2 & 0 & 2 & 6 \end{array}\right] \to \left[\begin{array}{cccc|c} 1 & -2 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 3 & 4 \end{array}\right] \to \left[\begin{array}{cccc|c} 1 & -2 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & -5 \end{array}\right]$

Not in column space.

(13) Let $C_1V_1 + C_2V_2 = 0$ $\Rightarrow \left[\begin{array}{cc|c} 1 & 2 & 0 \\ -1 & 2 & 0 \\ 0 & 1 & 0 \\ 2 & 4 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & 4 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$

$\Rightarrow C_1 = C_2 = 0$ $\Rightarrow$ linearly independent.

(15). Let $C_1V_1 + C_2V_2 + C_3V_3 = 0$

$\Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ -1 & 5 & -13 & 0 \\ 0 & 1 & -2 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & 7 & -14 & 0 \\ 0 & 1 & -2 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$\Rightarrow C_1 = -3C_3,\ C_2 = 2C_3$ $\Rightarrow$ linearly dependent. $-3\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + 2\begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix} + 1\begin{bmatrix} -1 \\ -13 \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$C_1 = -3$ $C_2 = 2$ $C_3 = 1$

(17) Let $C_1V_1 + C_2V_2 + C_3V_3 = 0$

$\cancel{\Rightarrow \left[\begin{array}{ccc|c} 2 & 1 & -4 & 0 \\ 5 & -1 & -10 & 0 \\ 3 & 1 & -6 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & \frac{1}{2} & -2 & 0 \\ 5 & \frac{1}{5} & -2 & 0 \\ 3 & \frac{1}{3} & -2 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & \frac{1}{2} & -2 & 0 \\ 0 & -\frac{27}{10} & 8 & 0 \\ 0 & -\frac{7}{6} & 4 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & \frac{1}{2} & -2 & 0 \\ 0 & -\frac{27}{10} & 8 & 0 \\ 0 & 0 & \frac{44}{81} & 0 \end{array}\right]}$

$\cancel{\Rightarrow C_1 = C_2 = C_3 = 0}$ ~~$\Rightarrow$ linearly independent.~~ $\underline{-2}\begin{bmatrix} 2 \\ 5 \\ 3 \end{bmatrix} \underline{+0}\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \underline{+1}\begin{bmatrix} -4 \\ -10 \\ -6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

linearly dependent

(18) $\begin{bmatrix} 1 & -3 \\ -1 & 3 \\ 1 & -3 \\ -1 & 3 \\ 1 & -3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -3 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \Rightarrow C(A) = \left\{ \begin{bmatrix} 1 \\ -3 \end{bmatrix} \right\}$

$\left[\begin{array}{ccccc|c} 1 & -1 & 1 & -1 & 1 & 0 \\ -3 & 3 & -3 & 3 & -3 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{ccccc|c} 1 & -1 & 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow X_1 = X_2 - X_3 + X_4 - X_5$

$\Rightarrow N(A) = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}.$

$\dim(C(A)) = 1, \quad \dim(N(A)) = 4.$

(21) $\begin{bmatrix} 2 & 4 & 1 & 1 \\ 1 & 2 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\Rightarrow$ Basis $= \left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \right\}$

(23) $\begin{bmatrix} 1 & -2 & 0 & 1 \\ 0 & 4 & 4 & 4 \\ 1 & 2 & 4 & 5 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -2 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 4 & 4 & 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -2 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\Rightarrow$ Basis $= \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 4 \\ 2 \end{bmatrix} \right\}$

(25) (a) T.

(b) F, $V_1 = \begin{bmatrix}1\\0\\0\\0\\0\\0\\0\end{bmatrix}$, $V_2 = \begin{bmatrix}0\\1\\0\\0\\0\\0\\0\end{bmatrix}$ $V_3 = \begin{bmatrix}0\\0\\1\\0\\0\\0\\0\end{bmatrix}$ $V_4 = \begin{bmatrix}0\\0\\0\\1\\0\\0\\0\end{bmatrix}$ $V_5 = \begin{bmatrix}0\\0\\0\\0\\1\\0\\0\end{bmatrix}$ $V_6 = \begin{bmatrix}0\\0\\0\\0\\0\\1\\0\end{bmatrix}$, $U = \mathbb{R}^7$

(c) F, $V_1 = V_2 = V_3 = V_4 = V_5 = V_6 = \begin{bmatrix}1\\0\end{bmatrix}$. $U = \mathbb{R}^2$

(d) F, $V_1 = V_2 = V_3 = V_4 = V_5 = V_6 = \begin{bmatrix}1\\0\end{bmatrix}$ $U = \mathbb{R}^2$

(e) F, $U = \mathbb{R}^2$, $V_1 = V_2 = V_3 = \begin{bmatrix}1\\0\end{bmatrix}$, $V_4 = V_5 = V_6 = \begin{bmatrix}0\\1\end{bmatrix}$

(f) T

(g) T

(h) T

(i) F. $U = \mathbb{R}^2$. $V_1 = V_2 = V_3 = \begin{bmatrix}1\\0\end{bmatrix}$, $V_4 = V_5 = V_6 = \begin{bmatrix}0\\1\end{bmatrix}$

(j) T

(27) $$A^T = \begin{bmatrix}1 & 1 & 1\\2 & 0 & 1\\3 & 1 & 1\\-2 & 0 & 1\\-1 & 1 & 1\\0 & 0 & 1\end{bmatrix} \Rightarrow \begin{bmatrix}1 & 1 & 1\\0 & -2 & -1\\0 & -2 & -2\\0 & 2 & 3\\0 & 2 & 2\\0 & 0 & 1\end{bmatrix} \Rightarrow \begin{bmatrix}1 & 1 & 0\\0 & -2 & 0\\0 & -2 & 0\\0 & 2 & 0\\0 & 2 & 0\\0 & 0 & 1\end{bmatrix} \Rightarrow \dim(C(A)) = 3$$

$$\left[\begin{array}{cccccc|c}1 & 2 & 3 & -2 & -1 & 0 & 0\\1 & 0 & 1 & 0 & 1 & 0 & 0\\1 & 1 & 1 & 1 & 1 & 1 & 0\end{array}\right] \Rightarrow \left[\begin{array}{cccccc|c}1 & 2 & 3 & -2 & -1 & 0 & 0\\0 & -2 & -2 & 2 & 2 & 0 & 0\\0 & -1 & -2 & 3 & 2 & 1 & 0\end{array}\right] \Rightarrow \left[\begin{array}{cccccc|c}1 & 2 & 3 & -2 & -1 & 0 & 0\\0 & 1 & 2 & -3 & -2 & -1 & 0\\0 & 0 & 2 & -4 & -2 & -2 & 0\end{array}\right]$$

$\Rightarrow$ 3 free variable $\Rightarrow \dim(N(A)) = 3$

(29) $A^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \\ 4 & 8 & 12 \\ 5 & 10 & 15 \\ 6 & 12 & 18 \\ 7 & 14 & 21 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $\dim(C(A^T)) = 1$

$$\left[\begin{array}{ccccccc|c} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 0 \\ 2 & 4 & 6 & 8 & 10 & 12 & 14 & 0 \\ 3 & 6 & 9 & 12 & 15 & 18 & 21 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{ccccccc|c} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow 6 \text{ free variable}$$

$\Rightarrow \dim(N(A)) = 6$

(31) $A = \begin{bmatrix} 0 & 1 & 2 & 1 \\ 0 & 1 & 1 & 2 \\ 0 & 1 & 2 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & 1 & 2 & 1 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \text{Basis}(C(A^T)) = \left\{ \begin{bmatrix} 0 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 2 \end{bmatrix} \right\}$

$$\left[\begin{array}{cccc|c} 0 & 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 1 & 2 & 1 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|c} 0 & 1 & 0 & 3 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \text{Basis}(N(A)) = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -3 \\ 1 \\ 1 \end{bmatrix} \right\}$$

(33) $\left[\begin{array}{ccccc|c} 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{ccccc|c} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{array}\right]$

$$\Rightarrow \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{array}\right] \Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -x_5 \\ x_5 \end{bmatrix}$$

(35). $\left[\begin{array}{ccccc|c} 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 2 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 2 & 1 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{ccccc|c} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{ccccc|c} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$

$\Rightarrow \left[\begin{array}{ccccc|c} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right] \Rightarrow. \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -x_3 \\ -x_3 \\ x_3 \\ 0 \\ 0 \end{bmatrix} = x_3 \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

(37). $\text{proj}_w v = \frac{w^T v}{w^T w} w = \frac{[1\ 0\ -2\ 0\ 2]\begin{bmatrix} 2 \\ 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}}{[1\ 0\ -2\ 0\ 2]\begin{bmatrix} 1 \\ 0 \\ -2 \\ 0 \\ 2 \end{bmatrix}} \cdot \begin{bmatrix} 1 \\ 0 \\ -2 \\ 0 \\ 2 \end{bmatrix} = -\frac{2}{9}\begin{bmatrix} 1 \\ 0 \\ -2 \\ 0 \\ 2 \end{bmatrix}$

(39) $P = \frac{ww^T}{w^T w} = \frac{1}{9} \cdot \begin{bmatrix} 2 \\ 1 \\ 0 \\ 2 \end{bmatrix} [2\ 1\ 0\ 2] = \frac{1}{9}\begin{bmatrix} 4 & 2 & 0 & 4 \\ 2 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 \\ 4 & 2 & 0 & 4 \end{bmatrix}$

(41). $A = \begin{bmatrix} 1 & 1 \\ 1 & 3 \\ 1 & 2 \end{bmatrix}. \quad b = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad A^T A \begin{bmatrix} C \\ D \end{bmatrix} = A^T b \Rightarrow \begin{bmatrix} 3 & 6 \\ 6 & 14 \end{bmatrix}\begin{bmatrix} C \\ D \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} C \\ D \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \\ 0 \end{bmatrix}$

(43). $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{bmatrix}. \quad b = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 2 \end{bmatrix}. \quad A^T A \begin{bmatrix} C \\ D \end{bmatrix} = A^T b \Rightarrow \begin{bmatrix} 4 & 10 \\ 10 & 30 \end{bmatrix}\begin{bmatrix} C \\ D \end{bmatrix} = \begin{bmatrix} 2 \\ 8 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} C \\ D \end{bmatrix} = \begin{bmatrix} -1 \\ \frac{3}{5} \end{bmatrix}$

(45) $a_1 = \frac{1}{3}\begin{bmatrix} 0 \\ 2 \\ -2 \\ 0 \\ 1 \end{bmatrix}$,

$$\begin{bmatrix} 1 \\ 5 \\ -3 \\ -1 \\ 2 \end{bmatrix} - \frac{\begin{bmatrix} 0 & 2 & -2 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 5 \\ -3 \\ -1 \\ 2 \end{bmatrix}}{\begin{bmatrix} 0 & 2 & -2 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 2 \\ -2 \\ 0 \\ 1 \end{bmatrix}}\begin{bmatrix} 0 \\ 2 \\ -2 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \\ 0 \end{bmatrix} \Rightarrow a_2 = \frac{1}{2}\begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 4 \\ -1 \\ -1 \\ -1 \end{bmatrix} - \frac{\begin{bmatrix} 0 & 2 & -2 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 4 \\ -1 \\ -1 \\ -1 \end{bmatrix}}{\begin{bmatrix} 0 & 2 & -2 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 2 \\ -2 \\ 0 \\ 1 \end{bmatrix}}\begin{bmatrix} 0 \\ 2 \\ -2 \\ 0 \\ 1 \end{bmatrix} - \frac{\begin{bmatrix} 1 & 5 & -3 & -1 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 4 \\ -1 \\ -1 \\ -1 \end{bmatrix}}{\begin{bmatrix} 1 & 5 & -3 & -1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 5 \\ -3 \\ -1 \\ 2 \end{bmatrix}}\begin{bmatrix} 1 \\ 5 \\ -3 \\ -1 \\ 2 \end{bmatrix} = \frac{1}{20}\begin{bmatrix} -11 \\ -15 \\ 53 \\ -9 \\ -62 \end{bmatrix}$$

$$\Rightarrow a_3 = \frac{1}{\sqrt{7080}}\begin{bmatrix} -11 \\ -15 \\ 53 \\ -9 \\ -62 \end{bmatrix}$$

(47). $A = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & -\frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 \end{bmatrix}$ $\quad P = A(A^TA)^{-1}A^T = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & \frac{3}{4} & -\frac{1}{4} & 0 & \frac{1}{4} & -\frac{1}{4} \\ 0 & \frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & -\frac{1}{4} & 0 & 0 \\ \frac{1}{4} & -\frac{1}{4} & 0 & -\frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & -\frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \end{bmatrix}$

(49) $\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 2 \\ 2 & 2 & 3 & 3 \\ 1 & -1 & 1 & -1 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & -2 & 0 & -2 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & -2 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & -2 & 0 & -2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{vmatrix} = 0 = 1\cdot(-2)\cdot 1\cdot 0$

(51). $\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 2 \\ 1 & 1 & 3 & 1 \\ 1 & 4 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 3 & 0 & 0 \end{vmatrix} = -\begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} = -1\cdot 3\cdot 2\cdot 1 = -6$

(53). $\det(A^{-1}) = \det^{-1}(A)$.

$$\det(A) = \begin{vmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 2 & 2 & 2 & 2 \\ 0 & 3 & 0 & 0 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 2 \\ 0 & 3 & 0 & 0 \end{vmatrix}. = -\begin{vmatrix} 1 & 1 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 1 & 0 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{vmatrix} = 1\cdot 3\cdot 1\cdot 2 = 6$$

$\det(A^{-1}) = \frac{1}{6}$

(55) $\begin{vmatrix} 1 & -2 & 1 & 0 \\ 0 & -2 & 0 & 1 \\ 0 & 0 & 0 & 5 \\ k & 2 & 0 & 3 \end{vmatrix} = \begin{vmatrix} 1 & -2 & 1 & 0 \\ 0 & -2 & 0 & 1 \\ 0 & 0 & 0 & 5 \\ 0 & 2+2k & -k & 3 \end{vmatrix} = \begin{vmatrix} 1 & -2 & 1 & 0 \\ 0 & -2 & 0 & 1 \\ 0 & 0 & 0 & 5 \\ 0 & 0 & -k & 4+k \end{vmatrix}$

$= -\begin{vmatrix} 1 & -2 & 1 & 0 \\ 0 & -2 & 0 & 1 \\ 0 & 0 & -k & 4+k \\ 0 & 0 & 0 & 5 \end{vmatrix} = -1\cdot(-2)(-k)\cdot 5 = 1 \Rightarrow k = -\frac{1}{10}$