Practice 1 for Midterm 1

Math 22A, Fall 2019

Name: ____

Student ID: __________________________________

You do not need to simplify numerical expressions for your final answers (e.g. you can write $3 - \frac{3}{4}$ instead of simplifying to $\frac{9}{4}$.)

Fill in all the blanks with your answers.

Problem 1: Calculate the vector given by the following linear combination

$$2\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} + 3\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

Answer: ______________

All linear combinations of

$$\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \text{ and } \begin{bmatrix} -2 \\ 14 \\ 4 \end{bmatrix} \text{ fill:}$$

A. A Line
B. A Plane
C. A Point
D. Three-dimensional space

Answer: ________

All linear combinations of

$$\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ -2 \\ -1 \end{bmatrix}, \text{ and } \begin{bmatrix} 3 \\ 6 \\ 3 \end{bmatrix} \text{ fill:}$$

A. A Line
B. A Plane
C. A Point
D. Three-dimensional space

Answer: ________

Problem 2: Determine whether the following vectors are perpendicular, parallel, or neither:

$$\begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix} \text{ and } \begin{bmatrix} -6 \\ 0 \\ 2 \end{bmatrix}$$

A. Perpendicular
B. Parallel
C. Neither

Answer: _________

$$\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \text{ and } \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$

A. Perpendicular
B. Parallel
C. Neither

Answer: _________

If θ is the angle between the following two vectors, calculate $\cos(\theta)$:

$$\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \text{ and } \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$$

$\cos(\theta)$: ___________

Multiply the following matrices with vectors, *or say "impossible" if they cannot be multiplied.*

$$\begin{bmatrix} -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \\ -2 \end{bmatrix}$$

Answer: _________

$$\begin{bmatrix} 1 & 3 \\ 2 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}$$

Answer: _________

Problem 3: Find a matrix filling in the blanks which switches the 2nd and 3rd rows:

$$\begin{bmatrix} \_ & \_ & \_ \\ \_ & \_ & \_ \\ \_ & \_ & \_ \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{bmatrix} a & b & c \\ g & h & i \\ d & e & f \end{bmatrix}$$

Write out the following system of equations in matrix form $Ax = b$:

$$\begin{aligned} x_1 - 2x_2 + x_3 &= 1 \\ x_1 - 2x_2 + 2x_3 &= 2 \\ 4x_2 - 2x_3 &= 3 \end{aligned}$$

$$\begin{bmatrix} \_ & \_ & \_ \\ \_ & \_ & \_ \\ \_ & \_ & \_ \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \_ \\ \_ \\ \_ \end{bmatrix}$$

Perform one row operation *on both sides* to modify the matrix equation so that the entries of the matrix in the 1^{st} column in the 2^{nd} and 3^{rd} rows are both 0 and record the result:

$$\begin{bmatrix} \_ & \_ & \_ \\ 0 & \_ & \_ \\ 0 & \_ & \_ \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \_ \\ \_ \\ \_ \end{bmatrix}$$

Perform another row operation *on both sides* to modify the matrix equation so that the entries of the matrix below the diagonal are all 0 and record the result:

$$\begin{bmatrix} \_ & \_ & \_ \\ 0 & \_ & \_ \\ 0 & 0 & \_ \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \_ \\ \_ \\ \_ \end{bmatrix}$$

Does the system of equations have

A. One solution
B. Infinitely many solutions
C. No solutions

Answer: __________

Problem 4: State whether or not the following matrices have an inverse or not:

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

A. Has an inverse
B. No inverse

Answer: __________

$$\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$$

A. Has an inverse
B. No inverse

Answer: __________

Calculate A^{-1} if A is the matrix

$$A = \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix}$$

Answer: ______________________________

Problem 5: Determine whether there is one solution, no solutions, or infinitely many solutions to $Ax = b$ if $A = LU$ where

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}, \; U = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad \text{and } b = \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}$$

A. One solution
B. Infinitely many solutions
C. No solutions

Answer: __________

Solve $Ax = b$ if $A = LU$ where

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}, \; U = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{bmatrix}, \quad \text{and } b = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$$

Answer: ______________________________