# MAT 022A MIDTERM 2 PRACTICE PROBLEMS

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(1) Prove that the subset of $\mathbb{R}^3$ of vectors $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ satisfying the equation $3x_1 + x_3 = 0$ is a subspace, by checking the two rules for a subspace hold.

(2) Prove that the subset of $\mathbb{R}^2$ of vectors $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ satisfying the equation $x_1 x_2 = 0$ is NOT a subspace, by GIVING AN EXAMPLE where at least one of the two rules for a subspace fails.

(3) Prove that the subset of $\mathbb{R}^3$ of vectors $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ satisfying the equation $x_1 + 2x_2 + x_3 = 5$ is NOT a subspace, by GIVING AN EXAMPLE where at least one of the two rules for a subspace fails.

(4) Prove that the subset of $\mathbb{R}^3$ of vectors $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ satisfying the equation $x_1 + x_2^2 = 0$ is NOT a subspace, by GIVING AN EXAMPLE where at least one of the two rules for a subspace fails.

(5) Is the column space of the following matrix a point, a line, a plane, or three-dimensional space?

$$A = \begin{bmatrix} 1 & 0 & -1 & 2 \\ 2 & 1 & -2 & 3 \\ -1 & 3 & 1 & -5 \end{bmatrix}$$

(6) Give an example of a matrix whose column space contains $(1, 2, 5)$ and $(0, 4, 1)$ and whose null space contains $(1, -1, 2)$.

(7) Give an example of a matrix whose null space contains $(1, 1, 0)$ and $(0, 2, 1)$.

(8) Describe the nullspace of the matrix $A$ as all linear combinations of a collection of two vectors (the answer should say what the two vectors are):

$$A = \begin{bmatrix} 1 & -2 & 1 & 3 \\ 2 & -4 & 1 & 1 \end{bmatrix}$$

(9) Describe the nullspace of the matrix $A$ as all linear combinations of a collection of two vectors (the answer should say what the two vectors are):

$$A = \begin{bmatrix} 0 & -1 & 2 & 3 \\ 1 & -3 & -1 & 2 \\ 1 & -4 & 1 & 5 \end{bmatrix}$$

(10) Describe all solutions to the equation $Ax = b$ when

$$A = \begin{bmatrix} 1 & 1 & -1 & -1 \\ 2 & 2 & 3 & 5 \\ 1 & 1 & 4 & 6 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 0 \\ 7 \\ 7 \end{bmatrix}$$

(11) Is the following vector $b$ in the column space of the given matrix $A$?

$$b = \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix} \quad \text{and} \quad A = \begin{bmatrix} 1 & -2 & 0 & -1 \\ 1 & -2 & 0 & 0 \\ 1 & -2 & 0 & 2 \end{bmatrix}$$

(12) Describe all solutions to the equation $Ax = b$ when

$$A = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 0 & -2 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 0 \\ 7 \\ 4 \end{bmatrix}$$

(13) State whether the following vectors are linearly independent or dependent. If they are linearly independent, prove it. If they are linearly dependent, give coefficients $c_1, c_2$ such that $c_1v_1 + c_2v_2 = 0$.

$$v_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 2 \\ 2 \\ 1 \\ 4 \end{bmatrix}$$

(14) State whether the following vectors are linearly independent or dependent. If they are linearly independent, prove it. If they are linearly dependent, give coefficients $c_1, c_2, c_3$ such that $c_1v_1 + c_2v_2 + c_3v_3 = 0$.

$$v_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

(15) State whether the following vectors are linearly independent or dependent. If they are linearly independent, prove it. If they are linearly dependent, give coefficients $c_1, c_2, c_3$ such that $c_1v_1 + c_2v_2 + c_3v_3 = 0$.

$$v_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix}, \quad v_3 = \begin{bmatrix} -1 \\ -13 \\ -2 \end{bmatrix}$$

(16) State whether the following vectors are linearly independent or dependent. If they are linearly independent, prove it. If they are linearly dependent, give coefficients $c_1, c_2, c_3, c_4$ such that $c_1v_1 + c_2v_2 + c_3v_3 + c_4v_4 = 0$.

$$v_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad v_4 = \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}$$

(17) State whether the following vectors are linearly independent or dependent. If they are linearly independent, prove it. If they are linearly dependent, give coefficients $c_1, c_2, c_3$ such that $c_1v_1 + c_2v_2 + c_3v_3 = 0$.

$$v_1 = \begin{bmatrix} 2 \\ 5 \\ 3 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \quad v_3 = \begin{bmatrix} -4 \\ -10 \\ -6 \end{bmatrix}$$

(18) Consider the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 3 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Find a basis for the column space $C(A)$. Find a basis for the nullspace $N(A)$. What is the dimension of $C(A)$? What is the dimension of $N(A)$? Check the rank nullity theorem holds in this case $(\dim(N(A)) + \dim(C(A)) = 4)$.

(19) Consider the matrix

$$A = \begin{bmatrix} 1 & -1 & 1 & -1 & 1 \\ -3 & 3 & -3 & 3 & -3 \end{bmatrix}$$

Find a basis for the column space $C(A)$. Find a basis for the nullspace $N(A)$. What is the dimension of $C(A)$? What is the dimension of $N(A)$? Check the rank nullity theorem holds in this case $(\dim(N(A)) + \dim(C(A)) = 5)$.

(20) Consider the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Find a basis for the column space $C(A)$. Find a basis for the nullspace $N(A)$. What is the dimension of $C(A)$? What is the dimension of $N(A)$? Check the rank nullity theorem holds in this case $(\dim(N(A)) + \dim(C(A)) = 3)$.

(21) Find a basis for the subspace $U$ given by

$$U = \text{Span}\left(\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}\right)$$

(22) Find a basis for the subspace $U$ given by

$$U = \text{Span}\left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ 4 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}\right)$$

(23) Find a basis for the subspace $U$ given by

$$U = \text{Span}\left(\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 4 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ 5 \end{bmatrix}\right)$$

(24) If $v_1, \cdots, v_5$ are 5 vectors in a subspace $U$, determine whether each statement is necessarily true or could sometimes (or always) be false. In cases where the statement is false, give an example showing it is false.

(a) $U$ has dimension 5.
(b) $U$ has dimension less than or equal to 5.
(c) $U$ has dimension greater than or equal to 5.
(d) If $U = \text{Span}(v_1, \cdots, v_5)$ then $(v_1, \cdots, v_5)$ are linearly independent.
(e) If $U = \text{Span}(v_1, \cdots, v_5)$ then $(v_1, \cdots, v_5)$ gives a basis for $U$.
(f) If $U = \text{Span}(v_1, \cdots, v_5)$ then $U$ has dimension less than or equal to 5.

(g) If $U = \text{Span}(v_1, \cdots, v_5)$ then $U$ has dimension greater than or equal to 5.
(h) If $U = \text{Span}(v_1, \cdots, v_5)$ then $U$ has dimension exactly 5.
(i) If $(v_1, \cdots, v_5)$ form a linearly independent list then $U = \text{Span}(v_1, \cdots, v_5)$.
(j) If $(v_1, \cdots, v_5)$ form a linearly independent list then $(v_1, \cdots, v_5)$ gives a basis for $U$.
(k) If $(v_1, \cdots, v_5)$ form a linearly independent list then $U$ has dimension less than or equal to 5.
(l) If $(v_1, \cdots, v_5)$ form a linearly independent list then $U$ has dimension greater than or equal to 5.
(m) If $(v_1, \cdots, v_5)$ form a linearly independent list then $U$ has dimension exactly 5.
(n) If $(v_1, \cdots, v_5)$ gives a basis for $U$ then $(v_1, \cdots, v_5)$ are linearly independent.
(o) If $(v_1, \cdots, v_5)$ gives a basis for $U$ then $U = \text{Span}(v_1, \cdots, v_5)$.
(p) If $(v_1, \cdots, v_5)$ gives a basis for $U$ then $U$ has dimension less than or equal to 5.
(q) If $(v_1, \cdots, v_5)$ gives a basis for $U$ then $U$ has dimension greater than or equal to 5.
(r) If $(v_1, \cdots, v_5)$ gives a basis for $U$ then $U$ has dimension exactly 5.
(s) If $U$ has dimension 5 then $(v_1, \cdots, v_5)$ are linearly independent.
(t) If $U$ has dimension 5 then $U = \text{Span}(v_1, \cdots, v_5)$.
(u) If $U$ has dimension 4 then $(v_1, \cdots, v_5)$ are linearly independent.
(v) If $U$ has dimension 4 then $(v_1, \cdots, v_5)$ span $U$.
(w) If $U$ has dimension 4 then $(v_1, \cdots, v_5)$ do not span $U$.
(x) If $U$ has dimension 6 then $(v_1, \cdots, v_5)$ are linearly independent.
(y) If $U$ has dimension 6 then $(v_1, \cdots, v_5)$ span $U$.
(z) If $U$ has dimension 6 then $(v_1, \cdots, v_5)$ do not span $U$.

(25) If $v_1, v_2, v_3, v_4, v_5, v_6$ are vectors in a subspace $U$, determine whether each statement is necessarily true or could sometimes (or always) be false. In cases where the statement is false, give an example showing it is false.

(a) If $(v_1, v_2, v_3, v_4, v_5, v_6)$ are linearly independent, then we can add 0 or more vectors to the list to get a basis for $U$.
(b) If $(v_1, v_2, v_3, v_4, v_5, v_6)$ are linearly independent, then we can remove 0 or more vectors to the list to get a basis for $U$.
(c) If $(v_1, v_2, v_3, v_4, v_5, v_6)$ are linearly dependent, then we can add 0 or more vectors to the list to get a basis for $U$.
(d) If $(v_1, v_2, v_3, v_4, v_5, v_6)$ are linearly dependent, then we can remove 0 or more vectors to the list to get a basis for $U$.
(e) If $U = \text{Span}(v_1, v_2, v_3, v_4, v_5, v_6)$, then we can add 0 or more vectors to the list to get a basis for $U$.
(f) If $U = \text{Span}(v_1, v_2, v_3, v_4, v_5, v_6)$, then we can remove 0 or more vectors to the list to get a basis for $U$.
(g) If $(v_1, v_2, v_3, v_4, v_5, v_6)$ are linearly independent and $U = \text{Span}(v_1, v_2, v_3, v_4, v_5, v_6)$, then we can add 0 or more vectors to the list to get a basis for $U$.
(h) If $(v_1, v_2, v_3, v_4, v_5, v_6)$ are linearly independent and $U = \text{Span}(v_1, v_2, v_3, v_4, v_5, v_6)$, then we can remove 0 or more vectors to the list to get a basis for $U$.
(i) If $(v_1, v_2, v_3, v_4, v_5, v_6)$ are linearly dependent and $U = \text{Span}(v_1, v_2, v_3, v_4, v_5, v_6)$, then we can add 0 or more vectors to the list to get a basis for $U$.
(j) If $(v_1, v_2, v_3, v_4, v_5, v_6)$ are linearly dependent and $U = \text{Span}(v_1, v_2, v_3, v_4, v_5, v_6)$, then we can remove 0 or more vectors to the list to get a basis for $U$.

(26) If

$$A = \begin{bmatrix} 12 & 5 & 3 & -7 & 0 & 1 & 13 \\ 101 & -6 & 15 & 3 & 3 & 1 & -8 \end{bmatrix}$$

what is the dimension of $C(A)$ and what is the dimension of $N(A)$?

(27) If

$$A = \begin{bmatrix} 1 & 2 & 3 & -2 & -1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

what is the dimension of $C(A)$ and what is the dimension of $N(A)$?

(28) If

$$A = \begin{bmatrix} 5 & 2 & 0 & -2 & -1 & 0 \\ 15 & 0 & 0 & 18 & 1 & 13 \\ -8 & 0 & 1 & 4 & 17 & -21 \end{bmatrix}$$

what is the dimension of $C(A)$ and what is the dimension of $N(A)$?

(29) If

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 4 & 6 & 8 & 10 & 12 & 14 \\ 3 & 6 & 9 & 12 & 15 & 18 & 21 \end{bmatrix}$$

what is the dimension of $C(A)$ and what is the dimension of $N(A)$?

(30) If

$$A = \begin{bmatrix} 1 & 0 & 3 & 4 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

Find a basis for $N(A)$ and a basis for $C(A^T)$. What are the dimensions of $N(A)$ and $C(A^T)$? Verify by direct computation using your bases that $N(A)$ is orthogonal to $C(A^T)$.

(31) If

$$A = \begin{bmatrix} 0 & 1 & 2 & 1 \\ 0 & 1 & 1 & 2 \\ 0 & 1 & 2 & 1 \end{bmatrix}$$

Find a basis for $N(A)$ and a basis for $C(A^T)$. What are the dimensions of $N(A)$ and $C(A^T)$? Verify by direct computation using your bases that $N(A)$ is orthogonal to $C(A^T)$.

(32) Find a non-zero vector which is perpendicular to the subspace

$$U = \text{Span}\left( \begin{bmatrix} 1 \\ 3 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 1 \\ 3 \end{bmatrix} \right)$$

(33) Find a non-zero vector which is perpendicular to the subspace

$$U = \text{Span}\left( \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right)$$

(34) Find a non-zero vector which is perpendicular to all the vectors in the following list:

$$v_1 = \begin{bmatrix} 1 \\ -1 \\ 2 \\ 0 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 3 \end{bmatrix}$$

(35) Find a non-zero vector which is perpendicular to all the vectors in the following list:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad v_4 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}$$

(36) Amongst the following subspaces, specify all pairs which are orthogonal to each other. Which of these pairs are orthogonal complements?

$$U_1 = \operatorname{Span}\left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right), \quad U_2 = \operatorname{Span}\left(\begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}\right), \quad U_3 = \operatorname{Span}\left(\begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}\right),$$

$$U_4 = \operatorname{Span}\left(\begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}\right), \quad U_5 = \operatorname{Span}\left(\begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix}\right), \quad U_5 = \operatorname{Span}\left(\begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix}\right)$$

(37) Calculate the projection of the vector $v$ onto the line $L = \operatorname{Span}(w)$ where

$$v = \begin{bmatrix} 2 \\ 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \quad w = \begin{bmatrix} 1 \\ 0 \\ -2 \\ 0 \\ 2 \end{bmatrix}$$

(38) Calculate the projection of the vector $v$ onto the line $L = \operatorname{Span}(w)$ where

$$v = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \quad w = \begin{bmatrix} 3 \\ 2 \\ -2 \\ 2 \\ 2 \end{bmatrix}$$

(39) Find the projection matrix which projects vectors onto the line $L = \operatorname{Span}(w)$ where

$$w = \begin{bmatrix} 2 \\ 1 \\ 0 \\ 2 \end{bmatrix}$$

(40) Find the projection matrix which projects vectors onto the line $L = \text{Span}(w)$ where

$$w = \begin{bmatrix} 3 \\ -1 \\ 1 \\ 2 \\ 1 \\ -3 \end{bmatrix}$$

(41) Calculate the line of best fit $y = C + Dx$ through the points $(1, 0)$, $(3, 0)$, $(2, 1)$.

(42) Calculate the line of best fit $y = C + Dx$ through the points $(1, 1)$, $(4, 0)$, $(2, 2)$.

(43) Calculate the line of best fit $y = C + Dx$ through the points $(1, 0)$, $(2, 0)$, $(3, 0)$, and $(4, 2)$.

(44) Find an orthonormal basis for the subspace

$$U = \text{Span}\left( \begin{bmatrix} 1 \\ 0 \\ -1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -1 \\ -2 \\ 3 \end{bmatrix} \right)$$

(45) Find an orthonormal basis for the subspace

$$U = \text{Span}\left( \begin{bmatrix} 0 \\ 2 \\ -2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 5 \\ -3 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ -1 \\ -1 \\ -1 \end{bmatrix} \right)$$

(46) Check that for the following vectors $(e_1, e_2, e_3)$ gives an orthonormal basis for $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} \frac{1}{\sqrt{6}} \\ \frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \end{bmatrix}, \quad v_2 = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ -\frac{1}{\sqrt{2}} \end{bmatrix}, \quad v_3 = \begin{bmatrix} \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{bmatrix}$$

Then solve the following matrix equation for $x, y, z$

$$\begin{bmatrix} \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \\ \frac{2}{\sqrt{6}} & 0 & -\frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

(47) Calculate the projection matrix which projects vectors onto the following subspace

$$U = \text{Span}\left( \begin{bmatrix} \frac{1}{2} \\ 0 \\ -\frac{1}{2} \\ 0 \\ \frac{1}{2} \\ 0 \\ \frac{1}{2} \end{bmatrix}, \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ 0 \\ 0 \\ \frac{1}{2} \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -\frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \\ 0 \\ 0 \end{bmatrix} \right)$$

(48) Calculate the projection matrix which projects vectors onto the following subspace

$$U = \text{Span}\left(\begin{bmatrix} \frac{2}{3} \\ 0 \\ \frac{2}{3} \\ \frac{1}{3} \\ 0 \end{bmatrix}, \begin{bmatrix} -\frac{1}{3} \\ 0 \\ 0 \\ \frac{2}{3} \\ -\frac{2}{3} \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -\frac{1}{3} \\ \frac{2}{3} \\ \frac{2}{3} \end{bmatrix}\right)$$

(49) Calculate the determinant of the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 2 \\ 2 & 2 & 3 & 3 \\ 1 & -1 & 1 & -1 \end{bmatrix}$$

(50) Calculate the determinant of the matrix

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 2 & 17 \\ 0 & 0 & 0 & 8 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

(51) Calculate the determinant of the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 2 \\ 1 & 1 & 3 & 1 \\ 1 & 4 & 1 & 1 \end{bmatrix}$$

(52) Calculate the determinant of the matrix

$$A = \begin{bmatrix} 0 & 3 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 2 & 0 \end{bmatrix}$$

(53) Calculate the determinant of $A^{-1}$ where $A$ is the matrix

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 2 & 2 & 2 & 2 \\ 0 & 3 & 0 & 0 \end{bmatrix}$$

(54) Determine the value of $k$ which ensures the following matrix has $\det(A) = 0$:

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 5 \\ 5 & 3 & k & 3 \end{bmatrix}$$

(55) Determine the value of $k$ which ensures the following matrix has $\det(A) = 1$:

$$A = \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & -2 & 0 & 1 \\ 0 & 0 & 0 & 5 \\ k & 2 & 0 & 3 \end{bmatrix}$$

(56) Determine the value of $k$ which ensures the following matrix has $\det(A) = 5$:

$$A = \begin{bmatrix} 1 & 1 & 2 & 2 \\ 1 & 1 & 1 & 2 \\ 1 & 2 & 2 & 2 \\ 1 & 1 & 2 & k \end{bmatrix}$$