MAT 022A CHAPTER 1,2 CUMULATIVE PRACTICE PROBLEMS

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(1) Let

$$A = \begin{bmatrix} 1 & 2 & 1 & -1 \\ 1 & 3 & 2 & 1 \\ 1 & 4 & 3 & 3 \end{bmatrix}$$

(a) Find a basis for $C(A)$, a basis for $N(A)$, the rank of A ($\dim(C(A))$) and the nullity ($\dim(N(A))$).
(b) Find a vector b such that $Ax = b$ has *no solutions*.
(c) Describe the general solution to $Ax = b$ when

$$b = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$$

(2) Let

$$A = \begin{bmatrix} 1 & -1 & 1 & -1 & 1 \\ 2 & -2 & 2 & 3 & 2 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 \end{bmatrix}$$

(a) Find a basis for $C(A)$, a basis for $N(A)$, the rank of A ($\dim(C(A))$) and the nullity ($\dim(N(A))$).
(b) Find a vector b such that $Ax = b$ has *no solutions*.
(c) Describe the general solution to $Ax = b$ when

$$b = \begin{bmatrix} 1 \\ -3 \\ 5 \\ 1 \end{bmatrix}$$

(3) Let

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 4 & 3 & 2 \\ 2 & 1 & 4 \end{bmatrix}$$

(a) Find the LU decomposition of A, i.e. find a lower triangular matrix L and an upper triangular matrix U so that $A = LU$ when
(b) Find A^{-1} or say if it does not exist.
(c) Find all solutions to $Ax = b$ when

$$b = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

(4) Let

$$A = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 3 & -1 \\ 3 & 3 & 5 \end{bmatrix}$$

(a) Find the LU decomposition of A, i.e. find a lower triangular matrix L and an upper triangular matrix U so that $A = LU$ when
(b) Find A^{-1} or say if it does not exist.
(c) Find all solutions to $Ax = b$ when

$$b = \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$$

(5) Let

$$A = \begin{bmatrix} 2 & 1 & 1 \\ -4 & -2 & -1 \\ 4 & 2 & 6 \end{bmatrix}$$

(a) Find the LU decomposition of A, i.e. find a lower triangular matrix L and an upper triangular matrix U so that $A = LU$ when
(b) Find A^{-1} or say if it does not exist.
(c) Find all solutions to $Ax = b$ when

$$b = \begin{bmatrix} 0 \\ 1 \\ 4 \end{bmatrix}$$

(6) Determine for each of the following subspaces of $\mathbb{R}^3$ whether it is a **point, line, plane, or all of** $\mathbb{R}^3$:
(a) The vectors $\langle x, y, z \rangle$ where $x + y + 2z = 0$.
(b) The subspace given as the span of the following vectors

$$U = \text{Span}\left(\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} -2 \\ -4 \\ 2 \end{bmatrix} \right)$$

(c) The vectors $\langle x, y, z \rangle$ where $x + 3y - z = 0$ and $x + 5z = 0$.
(d) The null space of the matrix

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

(e) The column space of the matrix

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

(f) The null space of the matrix

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(g) The column space of the matrix

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$