MAT 022A CHAPTER 6 PRACTICE PROBLEMS

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(1) Find all (real or complex) eigenvalues and a maximal collection of linearly independent eigenvectors for the following matrix. Can the matrix be diagonalized? If so give the factorization $A = XDX^{-1}$.

$$A = \begin{bmatrix} 4 & 5 \\ -1 & -2 \end{bmatrix}$$

(2) Find all (real or complex) eigenvalues and a maximal collection of linearly independent eigenvectors for the following matrix. Can the matrix be diagonalized? If so give the factorization $A = XDX^{-1}$.

$$A = \begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix}$$

(3) Find all (real or complex) eigenvalues and a maximal collection of linearly independent eigenvectors for the following matrix. Can the matrix be diagonalized? If so give the factorization $A = XDX^{-1}$.

$$A = \begin{bmatrix} -3 & 0 & 0 \\ -8 & 1 & 0 \\ 16 & -8 & -3 \end{bmatrix}$$

(4) Find all (real or complex) eigenvalues and a maximal collection of linearly independent eigenvectors for the following matrix. Can the matrix be diagonalized? If so give the factorization $A = XDX^{-1}$.

$$A = \begin{bmatrix} 2 & -1 \\ 2 & 4 \end{bmatrix}$$

(5) Find all (real or complex) eigenvalues and a maximal collection of linearly independent eigenvectors for the following matrix. Can the matrix be diagonalized? If so give the factorization $A = XDX^{-1}$.

$$A = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$$

(6) Find all (real or complex) eigenvalues and a maximal collection of linearly independent eigenvectors for the following matrix. Can the matrix be diagonalized? If so give the factorization $A = XDX^{-1}$.

$$A = \begin{bmatrix} 3 & 4 \\ -2 & -1 \end{bmatrix}$$

(7) Find all (real or complex) eigenvalues and a maximal collection of linearly independent eigenvectors for the following matrix. Can the matrix be diagonalized? If so give the factorization $A = XDX^{-1}$.

$$A = \begin{bmatrix} -5 & 0 & -4 \\ 20 & 2 & 11 \\ 6 & 0 & 5 \end{bmatrix}$$

(8) Find all (real or complex) eigenvalues and a maximal collection of linearly independent eigenvectors for the following matrix. Can the matrix be diagonalized? If so give the factorization $A = XDX^{-1}$.

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 2 & 0 \\ -1 & -14 & -2 \end{bmatrix}$$

(9) Find all (real or complex) eigenvalues and a maximal collection of linearly independent eigenvectors for the following matrix. Can the matrix be diagonalized? If so give the factorization $A = XDX^{-1}$.

$$A = \begin{bmatrix} 0 & 0 & -1 \\ 9 & 3 & 3 \\ 1 & 0 & 0 \end{bmatrix}$$

(10) True or False?

(a) Any $(n \times n)$ matrix has exactly n different eigenvalues.
(b) Any $(n \times n)$ matrix which has n different eigenvalues is symmetric.
(c) Any $(n \times n)$ matrix which has n linearly independent eigenvectors is diagonalizable.
(d) Any $(n \times n)$ matrix has at least n different eigenvalues.
(e) Any $(n \times n)$ matrix has exactly n linearly independent eigenvectors.
(f) Any $(n \times n)$ matrix which is diagonalizable is symmetric.
(g) Any $(n \times n)$ matrix has at least n linearly independent eigenvectors.
(h) Any $(n \times n)$ symmetric matrix has n linearly independent eigenvectors.
(i) Any $(n \times n)$ symmetric matrix is diagonalizable.
(j) Any $(n \times n)$ matrix which has n linearly independent eigenvectors has n different eigenvalues.
(k) Any $(n \times n)$ matrix has at most n linearly independent eigenvectors.
(l) Any $(n \times n)$ matrix which has n different eigenvalues has n linearly independent eigenvectors.
(m) Any $(n \times n)$ symmetric matrix has exactly n different eigenvalues.
(n) Any $(n \times n)$ matrix has at most n different eigenvalues.
(o) Any $(n \times n)$ matrix which has n different eigenvalues is diagonalizable.
(p) Any $(n \times n)$ matrix which is has n linearly independent eigenvectors is symmetric.

(11) For which value of c does the following matrix have exactly one real eigenvalue? Is the matrix with that value of c diagonalizable?

$$A = \begin{bmatrix} 5 & -2 \\ c & 1 \end{bmatrix}$$

(12) For which values of c does the following matrix have exactly two distinct real eigenvalues? For which values of c does the matrix have exactly one real eigenvalue? For which values of c does the matrix have exactly two complex eigenvalues? Find the eigenvalues and eigenvectors when $c = 0$. Find the eigenvalues and eigenvectors when $c = -2$. For what values of c is the matrix diagonalizable?

$$A = \begin{bmatrix} 3 & 3 \\ c & 5 \end{bmatrix}$$