# Optimal Transport and Placental Function

Qinglan Xia, Carolyn Salafia and Simon Morgan

**Abstract** The human newborn is a reflection of the entirety of nutrients transferred from the maternal to the fetal circulation across the placenta during gestation. By extension, birth weight and newborn health depend on placental function. The goal of this paper is to introduce the use of optimal transport modeling to study the expected effects of (i) placental size, (ii) placental shape (separate from size) and (iii) the position of insertion of the umbilical cord, on birth weight and placental functional efficiency. For each placenta (N=1110), a total transport cost based on all measurements (i), (ii), and (iii) is given by the model. This computed cost is highly correlated with measured birth weight, placenta weight, the fetal-placental weight ratio FPR, and the metabolic scaling factor beta. Next, a shape factor is calculated in a model of the total transport cost if each placenta were rescaled to have a unit area chorionic plate (thus separating shape from size). This shape factor is also highly correlated with birth weight, and after adjustment for placental weight, is highly correlated with the metabolic scaling factor beta.

## 1 INTRODUCTION

The human newborn is the reflection of the sum total of oxygen and nutrients transferred from the maternal to the fetal circulation across the placenta during gestation. By extension, birth weight depends on placental function. The goal of this paper is to apply optimal transport modeling to quantify effects of (i) placental size, (ii) placental shape and (iii) the position of insertion of the umbilical cord on the chorionic disk surface, on birth weight. This size, shape and position data was readily available from measurements from photographs of 1110 placentas from a University of North Carolina birth cohort collected in the middle of the last decade, which has been extensively studied in e.g. [9],[3] and references therein.

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The measures, (i),(ii) and (iii) above have expected effects on the energy required to pump blood across the placenta. Generally in any transport, and we assume also in the placenta, the less distance the blood has to travel, the less energy needs to be expended to pump it. Therefore the predicted optimum shape for the chorionic plate to minimize transportation energy is a circle with a centrally inserted umbilical cord. If the umbilical cord insertion point is eccentric within a circular chorionic plate then overall the blood will have farther to travel to and from to the umbilical cord. Also if the chorionic plate is not circular, but elliptical or lobated, then again, overall, the blood will have farther to travel and so more energy expenditure will be needed. Thus one may expect that placental shape and location of umbilical cord are important factors in determining the energy needed to pump blood across the fetal-placental circulation. From this, one would also assume that given a larger placenta, more blood would be transported over a longer distance, with more energy is required for pumping.

In this article we simulate a vascular tree structure for each placenta, in a simplified form by an idealized optimal transport network. For this network there is an associated total transport cost C, computed by the model. This cost C represents the total work done by the heart of fetus to pump blood across the placenta. We find a high correlation between C and measured birth weight, placenta weight, the fetal-placental weight ratio FPR and the metabolic scaling factor beta. Also a shape factor S is computed by the model which would be the total transport cost if a placenta were rescaled to have a unit area chorionic plate. This shape factor S is also highly correlated with birth weight, and after adjustment for placental weight, is highly correlated with the metabolic scaling factor beta.

## 2 Modeling Method

The optimal transportation problem aims at finding an optimal way to transport materials from the source to the target. An optimal transport path introduced in [5] is a mathematical concept used to model tree-shaped branching transport networks. Transport networks with branching structures are observable not only in nature as in trees, blood vessels, river channel networks, lightning, etc. but also in efficiently designed transport systems such as used in railway configurations and postage delivery networks. Recently, mathematicians (e.g.: [5] [4][2][1]) have shown great interest in modeling these transport networks with branching structures. Applications of optimal transport paths may be found in [6] and [7]. A related interesting approach is given in [10] which investigates thermodynamic properties of optimal transport networks while [11] investigates thermodynamic properties of measured human placenta major blood vessel networks. In this article, we will model the blood vessel structure of a placenta via an optimal transport path.

As stated in [9], 1110 placentas were collected by an academic health center in central North Carolina. For each placenta, a trained observer captured series of x,y coordinates that marked the site of the umbilical cord insertion and the perimeter of the fetal surface. To simulate vascular structures for the placentas, we apply the modeling method of ramified optimal transportation to each placenta.

An idealized transport network, which simulates an optimal vascular structure for that placenta, is computed based on the measurements of the placenta. This branched network provides a means of transporting blood between the whole chorionic plate surface and the umbilical cord. This single network for a placenta may be viewed as a representation of either an optimal vein network or, by reversing directions of flow, an optimal arterial network. In the absence of more detailed information about blood supply, we assume a uniform supply of blood per unit area over the whole surface of the placenta. We also model the placenta by a region in the plane because the data is from photographs of the placenta flat on a table, rather than in the curved inside surface of the uterus. The idealized transport network is a branched network of straight segments $e_i$ each with a capacity weighting $w_i$ and a direction of flow. For each branch point, the sum of flows in must equal the sum of flows out. Since there are many ways to construct a transport network we need to find an optimal network which minimizes the amount of work done in pumping blood through the network. In the model of ramified optimal transportation, we use the cost function $(w_i)^\alpha l_i$ for each edge $e_i$ of length $l_i$ where $\alpha$ is a branching parameter $(0 \leq \alpha < 1)$. Technically, as $x^\alpha$ is strictly concave for this range of $\alpha$, this ensures that branched structures will emerge and corresponds to the general principle of favoring transportation in groups and branched vessel structures. The total cost for each transport network, which reflects the work done to pump the blood, is the sum of the costs for each edge. Using algorithms stated in [8], for any fixed $\alpha$ we can build an approximating optimal transport path for a placenta using its measurements (e.g.: figure 1, left with $\alpha = 0.85$). Then we may calculate the associated total transport cost

$$C = \sum (w_i)^\alpha l_i$$

for that placenta.

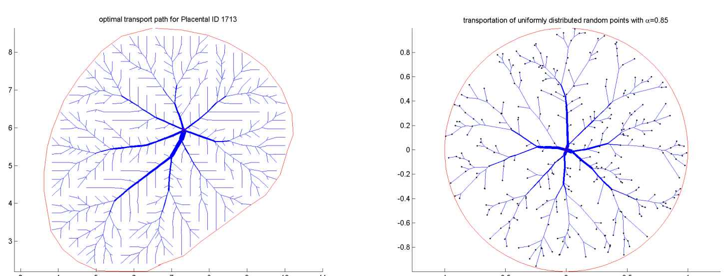


**Fig. 1** Examples of modeling blood vessels of a placenta by a nearly optimal transport network from the placenta surface to the umbilical cord. The distribution of the blood source over the surface of the placenta is uniform over lattice points of a fine regular grid in the example on the left, and is on randomly placed points in the example on the right.

For the calculations, we chose the value of $\alpha = 0.85$ so that, for a round placenta with a centrally inserted umbilical cord, 6 branches will emerge from the umbilical cord. This is consistent with the typical observation that 4 to 6 branches emerge from the umbilical cords in normal round placentas. We also used the uniformly distributed point sources as shown on the left of figure 1. This choice was made because if random distributions of point sources were used, the model would give different values of total transport cost $C$ for the same placenta each time the model was run. The total transport cost $C$ for each placenta depends upon shape, size and umbilical cord position. We want to investigate the effect of the shape and umbilical cord position independently from size. To do it, we consider

$$S = \frac{C}{A^{0.5+\alpha}},$$

where $A$ is the area of the placenta. Note that, the value of $S$ is a function of shape and cord position, and is independent of size. Indeed, suppose $D_1$ and $D_2$ are two placentas of the same shape. Then $D_1$ can be viewed as a rescale of $D_2$ with a length scaling factor $\lambda > 0$. Thus, $Area(D_1) = \lambda^2 Area(D_2)$. Let $G_1$ and $G_2$ be the corresponding optimal transport networks for $D_1$ and $D_2$. One may also show that the total cost $C_1$ for $G_1$ is the total cost $C_2$ for $G_2$ multiplied by $\lambda^{2\alpha+1}$. As a result,

$$S_1 = \frac{C_1}{A_1^{0.5+\alpha}} = \frac{\lambda^{2\alpha+1} C_2}{(\lambda^2 A_2)^{0.5+\alpha}} = \frac{C_2}{A_2^{0.5+\alpha}} = S_2.$$

We call $S$ the *shape factor* of the placenta. As a result, the total transport cost $C$ can be expressed as the product of two independent variables: $C = S * A^{0.5+\alpha}$.

## 3 RESULTS

For each of the 1110 placentas, the associated birth weight $B$ of the fetus and placental weight $P$ are also available. We applied the above method to calculate the total transport cost $C$ and the shape factor $S$. Placental functional efficiency is typically measured either by the fetal-placental weight ratio FPR= $\frac{B}{P}$ or by the metabolic scaling factor beta, $\beta = \frac{\ln B}{\ln P}$.

**Table 1** Pearson's Correlations

| | | Birth Weight | Placental Weight | FPR | beta |
|---|---|---|---|---|---|
| Total transport cost C | Pearson Correlation | .421 | .489 | -.154 | .272 |
| | Sig. (2 tailed) | .000 | .000 | .000 | .000 |
| Shape factor S | Pearson Correlation | -.080 | -.020 | -.056 | .039 |
| | Sig. (2 tailed) | .008 | .508 | .062 | .192 |

As shown in Table 1, total transport cost is highly correlated with birth weight, placental weight, FPR and beta. Total transport cost C is positively correlated with birth weight as expected given that C primarily reflects placental size, and on average will vary with larger and smaller placental and fetal weights.

On the other hand, the shape factor S is negatively correlated with birth weight as we would expect consistent with our hypothesis that a high S (and therefore an irregular shape with greater deviations of cord location and/or irregularities of perimeter) significantly impairs placental efficient for nutrient transportation under the conditions of an optimal transport network. In this sample the effect of shape factor S on birth weight is not paralleled by a correlation of abnormal shape with placental weight, with only trends to correlations with FPR and beta.

After adjustment for placental weight in regression analysis, the significant relationships of both total transport cost and the shape factor on birth weight remained (see Table 2). Both variables were also highly correlated with the metabolic scaling factor beta after adjustment for placental weight (see Table 3).

**Table 2** Regression Coefficients (point estimate of effect) for Total Transport Cost and Shape Factor on birth weight (Model 1) and after adjustment for placental weight (Model 2).

| Model | | Unstandardized Coefficients | | t | Sig. |
|---|---|---|---|---|---|
| | | Birth Weight | Std. Error | | |
| 1 | (Constant) | 2483.951 | 54.964 | 45.193 | .000 |
| | Total Transport Cost C | .590 | .038 | 15.400 | .000 |
| 2 | (Constant) | 1546.922 | 64.502 | 23.983 | .000 |
| | Total Transport Cost C | .210 | .037 | 5.639 | .000 |
| | Placental Weight | 3.307 | .158 | 20.958 | .000 |
| 1 | (Constant) | 3693.731 | 152.020 | 24.298 | .000 |
| | Shape Factor | -594.053 | 222.689 | -2.668 | .008 |
| 2 | (Constant) | 1985.163 | 134.301 | 14.781 | .000 |
| | Shape Factor | -501.411 | 173.258 | -2.894 | .004 |
| | Placental Weight | 3.734 | .139 | 26.837 | .000 |

For total transport cost, we do not expect model 2 to be greatly better than model 1, since placental area is factored into total transport cost and thus total transport cost in isolation includes placental size. However the shape factor S does not reflect placental size. Therefore we do expect the inclusion of placental weight into the model 2 to make a large difference as compared with model 1. The shape factor does not factor in placental area, and so does not reflect placental size. Model 1 includes shape factor S only, and thus no influence of placental size. Model 2 (which includes placental weight as a covariate) does. In both models shape factor S has a significant point estimate of effect on birth weight. The second model has a somewhat reduced point estimate of effect for shape factor S, with a smaller standard error, making this slightly smaller estimate of effect more precise.

Table 3 shows the same models of total transport cost and shape factor S predicting beta. Total transport cost is correlated with beta (placental functional efficiency). Model 2 includes placental weight; the distribution of beta varies with placental weight (heteroscedastic). Therefore, even though beta is calculated from placental weight, it is reasonable to include placental weight as a covariate. The point estimate of effect is reduced after adjustment for placental weight but remains highly statistically significant. Shape factor S is uncorrelated with beta in univariate regression, consistent with the results of correlation. Placental surface shape in isolation, out of context of other parameters of the placenta, would hardly be expected to be a

**Table 3** Regression Coefficients (point estimate of effect) for Total Transport Cost and Shape factor on beta (Model 1) and after adjustment for placental weight (Model 2).

| Model | | Unstandardized Coefficients | | t | Sig. |
|---|---|---|---|---|---|
| | | beta | Std. Error | | |
| 1 | (Constant) | .731 | .002 | 334.290 | .000 |
| | Total transport cost | 1.43E-005 | .000 | 9.374 | .000 |
| 2 | (Constant) | .680 | .002 | 324.002 | .000 |
| | Total transport cost | -6.2E-006 | .000 | -5.104 | .000 |
| | Placental weight | .000 | .000 | 34.606 | .000 |
| 1 | (Constant) | .743 | .006 | 130.033 | .000 |
| | Shape factor | .011 | .008 | 1.306 | .192 |
| 2 | (Constant) | .667 | .004 | 152.921 | .000 |
| | Shape factor | .015 | .006 | 2.670 | .008 |
| | Placental weight | .000 | .000 | 36.544 | .000 |

predictor of placental functional efficiency. However, the more regular the shape for a given placental weight (Model 2) the less the beta, and the larger the placenta relative to the birth weight (reflecting poorer functional efficiency). Thus while shape does not have independent effects on beta, the rounder any placenta is (the lower the shape factor S) at a given weight, the more efficient the placenta.

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