Errata for the book "Topics in Complex Analysis" by Dan Romik

Last updated: July 25, 2025

1. Page 85: replace the text (starting two lines below equation (2.6)): "Fix $\alpha > 0$. If s is in the half-plane [...] which, by the result of Exercise 1.26, is holomorphic in that region." with the following text:

   Fix a compact set $K \subset \{\mathrm{Re}(s) > 0\}$. Define numbers α, β by

   $$\alpha = \inf_{z \in K} \mathrm{Re}(s), \qquad \beta = \sup_{z \in K} \mathrm{Re}(s),$$

   and note that $0 < \alpha < \beta < \infty$. Then

   $$\left| \int_0^\infty e^{-x} x^{s-1}\, dx \right| \le \int_0^\infty e^{-x} \left| x^{s-1} \right| dx = \int_0^\infty e^{-x} x^{\mathrm{Re}(s)-1}\, dx \le \int_0^1 e^{-x} x^{\alpha-1}\, dx + \int_1^\infty e^{-x} x^{\beta-1}\, dx < \infty.$$

   Thus the improper integral (2.2) converges uniformly on compacts in $\{\mathrm{Re}(s) > 0\}$ and therefore defines a function $\Gamma(s)$ which, by the result of Exercise 1.26, is holomorphic in that region.

2. Page 242: in the proof of Lemma 6.6, change

   $$\text{“}\ |U(it)e^{\pi t z}| = O\left(t^2 e^{-\pi(\mathrm{Re}(z)+2)/t}\right) \qquad (t \to 0).\ \text{”}$$

   to:

   $$\text{“}\ |U(it)e^{\pi t z}| = O\left(t^2 e^{-\pi\,\mathrm{Re}(z) t}\right) \qquad (t \to 0).\ \text{”}$$

3. Page 242: in the proof of Lemma 6.6, change

   "$I_1(z) + I_2(z)$, where $I_1(z) = \int_0^1 U(\tau)e^{\pi i \tau z}\, d\tau$ and $I_1(z) = \int_1^\infty U(\tau)e^{\pi i \tau z}\, d\tau$,"

   to:

   "$I_1(z) + I_2(z)$, where $I_1(z) = \int_0^1 U(\tau)e^{\pi i \tau z}\, d\tau$ and $I_2(z) = \int_1^\infty U(\tau)e^{\pi i \tau z}\, d\tau$,"

4. Page 242: in the proof of Lemma 6.6, change

   "the improper integral $I_1(z)$ converges in the half-plane $\mathrm{Re}(z) > -2$ and defines a holomorphic function there. Similarly, $I_2(z)$ converges and is holomorphic in the half-plane $\mathrm{Re}(z) > 2$."

   to:

   "the improper integral $I_2(z)$ converges in the half-plane $\mathrm{Re}(z) > 2$ and defines a holomorphic function there. Similarly, $I_1(z)$ converges for all z and is an entire function."

5. Page 244: in the paragraph following equation (6.22), change:

   "It was established in that proof that this integral converges to a holomorphic function in the region $\mathrm{Re}(z) > -2$."

   to:

   "It was established in that proof that this integral converges to an entire function."

   In the next sentence, change "a holomorphic function, also in the region $\mathrm{Re}(z) > -2$" to "a holomorphic function, in this case in the region $\mathrm{Re}(z) > -2$"

6. Page 247: in equation (6.28), change the assumption "$\mathrm{Re}(z) > 3$" appearing in parentheses to "$\mathrm{Re}(z) > 3, |\,\mathrm{Im}(z)| < 1$".

7. Page 248: near the end of the proof of Lemma 6.11, change "so the bound (6.29)" to "so, if we now add the assumption that $|\,\mathrm{Im}(z)| < 1$, the bound (6.29)"

   Later on in the same sentence, change "implies a bound of the form (6.28) for $A(z)$" to "implies a bound of the form (6.28) for $A^{(k)}(z)$".

8. Page 249: in the two-line expression at the bottom of the page, the first summand

$$-i \int_{\Psi_1} U(\rho - 1)\rho^4 e^{\pi i \rho \|y\|^2} \frac{d\rho}{\rho^2}$$

   should be changed to:

$$-i \int_{\Psi_1} U(\rho - 1)\rho^2 e^{\pi i \rho \|y\|^2} \frac{d\rho}{\rho^2}$$

   and the second summand

$$-i \int_{\Psi_{-1}} U(\rho + 1)\rho^4 e^{\pi i \rho \|y\|^2} \frac{d\rho}{\rho^2}$$

   should be changed to:

$$-i \int_{\Psi_{-1}} U(\rho + 1)\rho^2 e^{\pi i \rho \|y\|^2} \frac{d\rho}{\rho^2}$$

9. Page 254: in the mathematical display in Lemma 6.22, change the assumption "$\mathrm{Re}(z) > 3$" appearing in parentheses to "$\mathrm{Re}(z) > 3, |\,\mathrm{Im}(z)| < 1$".

10. Page 263: in the statement of Lemma 6.31, change "in the region $\mathrm{Re}(z) > -2$" to "in the region $\mathrm{Re}(z) > -1$".

11. Page 264: in the two-line display on the bottom half of the page, change the second line

$$\text{“ } \widehat{\varphi}(0) = \varphi_+(0) - \varphi_+(0) = 240\pi, \text{ ”}$$

   to:

$$\text{“ } \widehat{\varphi}(0) = \varphi_+(0) - \varphi_-(0) = 240\pi, \text{ ”}$$

12. Page 282: in the statement of Lemma A.28, change

   "If $f : \mathbb{R}^d \to \mathbb{R}$ is a Schwartz function that is a magic function for a lattice $\Lambda \subset \mathbb{R}^d$,"

   to:

   "If $f : \mathbb{R}^d \to \mathbb{R}$ is a Schwartz function that is a magic function for a lattice $\Lambda \subset \mathbb{R}^d$ with $\mathrm{covol}(\Lambda) = 1$,"

13. Pages 282–283: replace the proof of Lemma A.28 with the following text:

Combining the Poisson summation formula (A.6) with the assumptions on f and Λ, we have that

$$\begin{aligned} f(0) &\geq f(0) + \sum_{x \in \Lambda \setminus \{0\}} f(x) = \sum_{x \in \Lambda} f(x) \\ &= \sum_{y \in \Lambda^*} \widehat{f}(y) = \widehat{f}(0) + \sum_{y \in \Lambda^* \setminus \{0\}} \widehat{f}(y) \geq \widehat{f}(0) = f(0). \end{aligned}$$

Since this chain of inequalities starts and ends with $f(0)$, both of the (weak) inequalities in the chain actually hold as *equalities*. The only way in which this can be true is if all the summation terms that were discarded to obtain those inequalities—the terms $f(x)$ for $x \in \Lambda \setminus \{0\}$ in the first inequality, which were known to be nonpositive, and the terms $\widehat{f}(y)$ for $y \in \Lambda^* \setminus \{0\}$ in the second inequality, which were known to be nonnegative—are necessarily 0; this was the claim to be proved. □