Math 205A: Complex Analysis
Course Syllabus
UC Davis, Winter 2018

Dan Romik*

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1 Course meeting information

- Course lectures: MWF 1:10-2:00, Bainer 1134
- Office hours: F 10:30-11:30, MSB 2218.

2 Prerequisites

Undergraduate complex analysis (UC Davis Math 185, or equivalent).

3 Course textbook

- The course will be based to a large extent on the book *Complex Analysis*, by E. M. Stein and R. Shakarchi (Princeton University Press, 2003).

*For questions about the class, email me at `romik@math.ucdavis.edu`.

- The textbook will be supplemented by my own text, *Complex Analysis Lecture Notes*, which I wrote in 2016 and will be continuing to improve over time. The lecture notes are available to download from the course webpage.
- Additional recommended reading will be suggested for some of the topics covered.

4 Course description and learning objectives

The course is the first in the two-quarter graduate sequence in complex analysis. The course aims to revisit the material from undergraduate complex analysis and a select number of more advanced topics, emphasizing the beauty of the theory and its applicability and connections to other areas of mathematics.

Learning objectives

1. To relearn the material from undergraduate complex analysis at a higher level of rigor and depth.
2. To learn a select number of more advanced topics, including both theory and applications.
3. To improve your general abilities as a pure mathematician, including:
    - Proof writing and mathematical exposition skills
    - Proof reading and critiquing skills
    - Understanding of the mathematical analyst's way of thinking, e.g., ϵ-δ arguments, manipulation of inequalities.
    - Optional, but strongly recommended: mathematical typesetting (i.e., LaTeX) skills.

Detailed list of topics: [estimated class time in brackets]

- The fundamental theorem of algebra: three "proofs from the book" [**1 lecture**]

- Basic complex analysis: differentiation, analytic and harmonic functions, the Cauchy-Riemann equations, power series, the exponential and trigonometric functions, conformality, the Riemann sphere, stereographic projection [**3 lectures**]
- Integration and Cauchy's theorem: topology of planar curves, proof of Cauchy's theorem, the Cauchy integral formulas [**4 lectures**]
- Applications of Cauchy's theorem: the logarithm function, Liouville's theorem, the maximum principle, Rouché's theorem, the argument principle, principle of analytic continuation [**3 lectures**]
- Meromorphic functions; the residue theorem and applications [**2 lecture**]
- The Euler gamma function and its properties [**2 lecture**]
- The Riemann zeta function and its properties [**2 lectures**]
- Asymptotic analysis, the saddle-point method, Stirling's formula, the prime number theorem. [**3 lectures**]
- Entire functions of finite order, Hadamard's factorization theorem [**3 lectures**]
- Additional topics as time permits, such as: conformal maps and the Riemann mapping theorem; the Dirichlet problem and other applications to partial differential equations; doubly-periodic functions; the Mellin transform; ....

5 Grading policy

The course grade will be assigned based on two homework assignments (25%) and an in-class final exam (75%), with an extra credit option (8%).

- **Homework:** homework will be assigned every week but not graded, except for two of the weekly homework assignments. Those mandatory assignments will be graded and critiqued in detail by me for correctness

and the quality of the presentation, and will count for 25% of the final course grade, with the higher-graded assignment counting for 15% and the lower-graded one counting for 10%.

- **Exam:** the final exam will count for 75% of the final course grade.
- **Extra credit:** up to 8% extra credit will be given for helping me improve the course lecture notes with feedback or other contributions, for example: pointing out typos or errors, simplifying arguments in the proof, suggesting cool material to include, etc. (If you really want to distinguish yourself, try creating nice graphic figures to illustrate some of the proofs, or doing something equally impressive/creative like writing a whole new section on an interesting subtopic.) The amount of extra credit will depend on the quantity and quality of the feedback you send me. I will also acknowledge any contributions in the lecture notes.

6 Ethics policy

Any work submitted as part of the homework assignments must: (i) be physically written/typed by you; (ii) be written in your own words; and (iii) represent that you have taken a significant intellectual part in its creation and completely understand what you have written, unless explicitly specified otherwise. (I.e., you may work on a problem in collaboration with peers as long as you make a sincere effort to solve it yourself, and once a solution has been found by the group you must make sure that you understand it completely if you are submitting it as part of the assignment, or explicitly clarify which part you are not sure you understand.)

Failure to adhere to these guidelines would be considered by me as a violation of the UC Davis Code of Academic Conduct and warrant, at minimum, a failing grade in the assignment in question and a referral to Student Judicial Affairs.