Practice problem set for Midterm Exam 1

MAT 21B (UC Davis, Winter 2018)

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Notes — please read carefully:

- The first midterm exam will be held on Wednesday 1/31 in the regular lecture hall (Giedt 1001) at the regular class meeting time (W 5:10-6)
- The midterm will be 50 minutes long.
- No calculators, books, or notes will be allowed in the exam.
- The exam will cover Sections 4.8, 5.1–5.5 in the book Thomas' Calculus (13th Ed.).
- The problems below are offered as a supplementary aid to help you challenge yourself and test your understanding of the material. No attempt has been made to cover all the possible topics that may be asked about on the exam, or to ensure that the difficulty level of the problems below matches the eventual difficulty of the midterm questions.
- Solutions to this problem set will be posted online in the next few days.
- **Very important guidance:** In this and all future 21B exams, points will be awarded for a correct final answer accompanied by the work showing how you arrived at the answer. That is:
  - If you do not show your work, you will not get *any* points regardless of whether the final answer is correct.
  - If you show your work and it is essentially correct but the final answer is incorrect due to a calculation error, partial credit will be awarded (most of the points, in the case of small errors like a flipped sign).
  - To determine that you have showed your work, the graders will consider whether the work is readable and intelligible to them, so you should make an effort to effectively *communicate* your work in a clean, readable, and syntactically and grammatically correct form. You are recommended to work on this (often neglected) aspect of problem-solving already while you are working on the practice problems.

1 Write the sum $1 + 3 + 5 + 7 + \ldots + 97 + 99$ in $\sum$ notation, and evaluate it. You may use the formula $\sum_{m=1}^{N} m = N(N+1)/2$.

2 (a) Evaluate the definite integral $A = \int_1^{10} \frac{1}{x}\, dx$. (An answer expressed in terms of values of standard mathematical functions is acceptable.)

(b) Define $B = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{9}$. Without using a calculator, determine which of A and B is larger, and explain how you can tell.

(c) Define $C = B - 1 + \frac{1}{10}$. Without using a calculator, determine which of A and C is larger, and explain how you can tell.

3 (a) Compute the indefinite integral $\int 10\cos(x)\sin^4(x)\, dx$.

(b) Find an antiderivative for $f(x) = \frac{1}{(x-2)^4}$.

(c) Find *all* antiderivatives $F(x)$ for $f(x) = 2\sin x \cos x$ that satisfy $F(0) = 3$.

4 Use the midpoint rule and a subdivision of the interval $[-1, 2]$ into 3 subintervals of equal lengths to compute a numerical approximation to the definite integral $\int_{-1}^{2} |x|\, dx$.

5 The temperature in Davis on the first week of January 2018 was measured to be (in Fahrenheit degrees) approximately

$$f(t) = 53 + 10\sin(2\pi t), \qquad 1 \le t \le 8,$$

where we model the week as the interval $[1, 8]$, with the subinterval $[1, 2]$ corresponding to Monday, $[2, 3]$ corresponding to Tuesday, etc. What was the average temperature in Davis that week?

6 According to data published on the website zeroto60times.com, a 2017 Tesla Model S 60 car accelerates from 0 to 60 mph in 5 seconds. (Note that a speed of 60 mph is equivalent to 88 feet per second.) Assume that the acceleration is uniform over that 5-second interval. How many feet has the car traveled at the moment it hits 60 mph?

7 If $f(x)$ is integrable and $\int_0^4 f(x)\, dx = 10$, $\int_2^4 f(x)\, dx = 6$, what is $\int_0^1 f(2x)\, dx$?

8 (a) Write a Riemann sum S_n with n summands for $f(x) = x - 1$ in the interval $[2, 6]$.

(b) Calculate the limit of S_n as $n \to \infty$ to obtain the value of $\int_2^6 f(x)\, dx$. (You may find the formula $\sum_{m=1}^{N} m = N(N+1)/2$ helpful for this purpose.)

9 (a) Define a function $f(n)$ of an integer $n \geq 1$ by $f(n) = \sum_{k=1}^{n} (-1)^{k+1} k^2$. Compute the values $f(1), f(2), f(3), f(4), f(5), f(6)$.

(b) Similarly, define two functions $g(n)$ and $h(n)$ of an integer $n \geq 1$ by $g(n) = \sum_{k=1}^{n} k^2$ and $h(n) = \frac{1}{6} n(n+1)(2n+1)$. Show that

$$g(n) - g(n-1) = n^2 \qquad \text{and} \qquad h(n) - h(n-1) = n^2.$$

(Note: the fact that both $g(n)$ and $h(n)$ satisfy this equation can be thought of as a kind of "sanity check" that makes it plausible to believe that $g(n) = h(n)$ for all n. It also makes it possible to prove this using the proof method known as mathematical induction.)

10 Evaluate the following definite integrals:

(a) $\int_{-1}^{3} \sqrt{1+x}\, dx$

(b) $\int_{0}^{2} e^{x^3} x^2\, dx$ (a non-numerical answer in terms of e or other standard mathematical constants or functions is acceptable)