Instructions. The homework is due by **Wednesday 1/15/20**. Please upload the homework electronically through the Assignments folder on the course Canvas site. On the main Canvas page, click on the Assignments folder and once this page is open, click on the assignment Homework 1. There will be a button for you to click on to upload your assignment. If you typed up your homework, please only upload the final pdf file. If you hand-wrote your work, either scan or take clear pictures of your work and upload these files. Make sure that all solutions you wrote are present and easily readable when your assignment is handed in as late homework is not accepted.

Reading material. Read Chapters 1–2 in the textbook.

Problems

1. For each of the following systems of linear equations, if the system has a unique solution, find it; otherwise, indicate if there are no solutions or an infinite number of solutions.

   (a) $\begin{cases} x + y = 22 \\ x - 2y = 7 \end{cases}$

   (b) $\begin{cases} -4x + 2y = 10 \\ 2x - y = 10 \end{cases}$

   (c) $\begin{cases} x + y = 10 \\ 5x + 5y = 50 \end{cases}$

   (d) $\begin{cases} x + 2y + 3z = 11 \\ 2x + 6z = 14 \\ x + y + z = 5 \end{cases}$

   For (d), consider referring to Appendix A (particularly section A 3.1–A 3.3) for strategies for solving linear systems with more than two equations.

2. In the story believed to be the origin of the expression "*Eureka!*", the Greek mathematician Archimedes discovered how to measure the volume of an object by immersing it in liquid, and in doing so helping King Hiero II of Syracuse to find out if the king's goldsmith had cheated him by using a higher percentage of silver and a smaller one of gold than he was supposed to use in a crown he had been commissioned to make for the king.

   Assume that the crown was made of a mixture of gold and silver, weighed 1 pound (equal to about 454 grams) and had a volume of 30 cubic centimeters. The density (mass per unit volume) of gold is known to be 19.3 grams per cubic centimeter, and the density of silver is 10.5 grams per cubic centimeter. What percentage of the crown's weight is gold?

3. Solve the following exercises in the textbook:

   (a) Chapter 1: "Proof-writing exercise" number 1.
   (b) Chapter 2: "Calculational exercises" number 1, 2, 3, 4(a), 4(b), 5(c), 5(d).

(c) Chapter 2: "Proof-writing exercises" number 2, 3.

Remarks on writing proofs: 1. when you are asked to prove something, simply try to write a logical and clear verbal explanation of why the claim is true. In some cases the thing you are being asked to prove may appear obvious; that means you need to pause and think *why* it's obvious — i.e., what assumptions you are making (perhaps subconsciously) that make it appear "obviously" true — and write what those assumptions are and how they relate to the claim.

2. To prove a statement of the form "A is true if and only if B is true", prove the two statements "If A is true then B is true" and "If B is true then A is true".