

Homework due: Wednesday 1/22/20 by uploading your solution to Canvas in the Assignments module.

Reading material. Read Sections 3.2–4.3 in the textbook (the proof of the Fundamental Theorem of Algebra in section 3.1 is optional material).

Problems

1. In an earlier lecture we encountered the (non-linear) system of equations

$$\begin{cases} x + y = 1 \\ 5x + 2y = zx \\ 2x + 8y = zy \end{cases}$$

which came up in the application of the Google PageRank algorithm. In that example, in order to solve for x, y we had to provide the value of z without explaining how it was obtained. This problem provides that missing explanation.

- (a) Show how the second and third equations in the system lead to a quadratic equation whose two solutions are the possible values of z , by applying the criterion in “proof-writing exercise” 1 of Chapter 1 in the textbook.
 - (b) For each of the two values of z obtained in part (a), solve the system for x and y , and explain why only one of the solutions makes sense (in terms of the meaning of x and y in the original problem).
2. For each of the following pairs of polynomials $f(x)$ and $g(x)$, perform polynomial long division with remainder of $f(x)$ by $g(x)$. That is, find polynomials $q(x)$ (the “quotient”) and $r(x)$ (the “remainder”) such that $\deg r < \deg g$ and

$$f(x) = q(x)g(x) + r(x).$$

- (a) $f(x) = x^3 - x^2 + 2$, $g(x) = x - 2$
 - (b) $f(x) = x^3 - x^2 + 2$, $g(x) = x + 1$
 - (c) $f(x) = x^4 + x$, $g(x) = x^2 + 1$
3. You are given the polynomial $p(x) = x^3 - 4x^2 + 2x + 3$ together with the information that $p(3) = 0$. Find its other two roots.

Hint: Reduce the problem to a quadratic equation by dividing $p(x)$ by an appropriate factor.

4. Let n be a positive integer. Find the quotient of the polynomial $f(x) = x^n - 1$ by the polynomial $g(x) = x - 1$ (in this case there is no remainder since $f(1) = 0$). It is recommended to solve this first for small values $n = 2, 3, \dots$ of n and then generalize the solution.
5. Solve the following problems in the textbook:
 - (a) Computational exercise 1 in Chapter 3.

- (b) Proof-writing exercise 2(a) in Chapter 3.
- (c) Computational exercise 1 in Chapter 4.
- (d) Proof-writing exercise 1 in Chapter 4.