**Homework due.**  Wednesday 2/19/20 via upload to Canvas.

**Reading material.**  Chapter 6 in the textbook.

**Problems**

1. Solve the following problems in the textbook:
   
   (a) Calculation exercises 1(b), 1(f), 2(a), 6 in Chapter 6.
   
   (b) Proof-writing exercise 2 in Chapter 6.

2. Compute the coordinate vector $[v]_B$, where:
   
   (a) $v = (1, 0, 1), B = \{(1, 0, 0), (1, 1, 0), (1, 1, 1)\}$.
   
   (b) $v = (1, 0, 1), B = \{(1, 0, 1), (1, 0, 0), (0, 1, 0)\}$.
   
   (c) $v = z^3 - 2z, B = \{z + 1, z - 1, z^2, z^3\}$ in the space $P_3$ of polynomials of degree $\leq 3$.

3. Compute the representation matrix $M = M(T)^B_C$ of a linear transformation $T$ relative to two bases $B, C$, where:
   
   (a) $T(x, y) = (x + 10y, -x), B = C = \{(1, 0), (0, 1)\}$.
   
   (b) $T(x, y, z) = (z, y, 3x), B = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}, C = \{(1, 0, 1), (0, 1, 0), (-1, 0, 1)\}$
   
   (c) $T(p) = p'$ (the derivative operator) considered as a linear map from the space $P_2$ of polynomials of degree $\leq 2$ (aka quadratic polynomials) to the space $P_1$ of polynomials of degree $\leq 1$ (aka linear functions), with $B = \{1, x, x^2\}, C = \{1, x\}$.

4. Let $U, V, W$ be finite-dimensional vector spaces, and let $S : U \to V, T : V \to W$ be linear transformations. The goal of this problem is to prove the inequality:

$$\dim(\text{null}(T \circ S)) \leq \dim(\text{null}(S)) + \dim(\text{null}(T)),$$

where $T \circ S : U \to W$ denotes the composition of the two transformations. Prove this by using the following steps:

   (a) Denote $H = \text{null}(T \circ S)$ (a linear subspace of $U$), and define a linear transformation $R : H \to V$ by $R(v) = S(v)$ (i.e., it is the same as $S$, but its domain is a subspace of the domain of $S$; sometimes $R$ defined in this way will be referred to as the \textit{restriction of $S$ to $H$}). Show that $\text{null}(S) \subseteq H$, and explain why this implies that $\text{null}(R) = \text{null}(S)$.

   (b) Show that $\text{range}(R) \subseteq \text{null}(T)$.

   (c) Apply the dimension formula (Theorem 6.5.1 in the textbook) for a suitable linear transformation to deduce the inequality stated at the beginning of the question.