Homework due. Wednesday 2/19/20 via upload to Canvas.

Reading material. Chapter 6 in the textbook.

Problems

1. Solve the following problems in the textbook:
   (a) Calculation exercises 1(b), 1(f), 2(a), 6 in Chapter 6.
   (b) Proof-writing exercise 2 in Chapter 6.

2. Compute the coordinate vector $[v]_B$, where:
   (a) $v = (1, 0, 1)$, $B = \{(1, 0, 0), (1, 1, 0), (1, 1, 1)\}$.
   (b) $v = (1, 0, 1)$, $B = \{(1, 0, 1), (1, 0, 0), (0, 1, 0)\}$.
   (c) $v = z^3 - 2z$, $B = \{z + 1, z - 1, z^2, z^3\}$ in the space P_3 of polynomials of degree ≤ 3.

3. Compute the representation matrix $M = M(T)^B_C$ of a linear transformation T relative to two bases B, C, where:
   (a) $T(x, y) = (x + 10y, -x)$, $B = C = \{(1, 0), (0, 1)\}$.
   (b) $T(x, y, z) = (z, y, 3x)$, $B = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$, $C = \{(1, 0, 1), (0, 1, 0), (-1, 0, 1)\}$
   (c) $T(p) = p'$ (the derivative operator) considered as a linear map from the space P_2 of polynomials of degree ≤ 2 (aka quadratic polynomials) to the space P_1 of polynomials of degree ≤ 1 (aka linear functions), with $B = \{1, x, x^2\}$, $C = \{1, x\}$.

4. Let U, V, W be finite-dimensional vector spaces, and let $S : U \to V$, $T : V \to W$ be linear transformations. The goal of this problem is to prove the inequality:

$$\dim(\text{null}(T \circ S)) \leq \dim(\text{null}(S)) + \dim(\text{null}(T)),$$

   where $T \circ S : U \to W$ denotes the composition of the two transformations. Prove this by using the following steps:
   (a) Denote $H = \text{null}(T \circ S)$ (a linear subspace of U), and define a linear transformation $R : H \to V$ by $R(v) = S(v)$ (i.e., it is the same as S, but its domain is a subspace of the domain of S; sometimes R defined in this way will be referred to as the *restriction of S to H*). Show that $\text{null}(S) \subseteq H$, and explain why this implies that $\text{null}(R) = \text{null}(S)$.
   (b) Show that $\text{range}(R) \subseteq \text{null}(T)$.
   (c) Apply the dimension formula (Theorem 6.5.1 in the textbook) for a suitable linear transformation to deduce the inequality stated at the beginning of the question.