Homework due. Wednesday 3/4/20 via upload to Canvas.

Reading material. Chapters 1–8 in the textbook.

Problems

1. The final exam is coming up in less than 3 weeks. Consider it one of your homework assignments for this week to spend a decent amount of time (say, 1–2 hours) reviewing the material so far, solving and reviewing solutions for homework problems you did not yet master, and shoring up your understanding of basic topics.

2. Solve calculational problems 4, 5 in Chapter 8 of the textbook.

3. For each of the following matrices, find all (complex) eigenvalues of the matrix, and find a basis of eigenvectors (allowing also vectors with complex coordinates) if such a basis exists.

   (a) $\begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix}$

   (b) $\begin{pmatrix} 1 & 3 \\ 0 & 2 \end{pmatrix}$

   (c) $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$

   (c) $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

   (d) $\begin{pmatrix} 0 & 2 & 2 \\ 2 & 0 & 2 \\ 2 & 2 & 0 \end{pmatrix}$

   (e)* (optional) $\overbrace{\begin{pmatrix} a & b & b & b & \cdots & b \\ b & a & b & b & \cdots & b \\ b & b & a & b & \cdots & b \\ b & b & b & a & \cdots & b \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ b & b & b & b & \cdots & a \end{pmatrix}}^{n \text{ rows, } n \text{ columns}}$ (for arbitrary numbers a, b)

   Hint for part (e). Consider separately the cases $b = 0$ and $b \neq 0$. For the case $b \neq 0$, show that the problem can be reduced to the case $a = 1, b = 1$, then work directly with the equation $Av = \lambda v$ to identify the possible solutions in that case.

4. Explain how the example illustrating the PageRank search engine algorithm from Lecture 1 can be reformulated as a question about an eigenvector of a certain matrix. What is the matrix? What is the relevant eigenvalue, and how did we identify it?