helpful. We have been successful with the proposed rejection formula, both in cases when no object was present in the key image (i.e., it contained only background) and when the image contained unknown or unlearned objects.

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Generalized *E*-Filter and Its Application to Edge Detection

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Abstract—**The *E*-filter is a homomorphic filter proposed by Moore and Parker which differentiates the small and large amplitude components of a signal. In this correspondence, we describe a generalization of the *E*-filter to include a control parameter. Selectively tuning this parameter causes the behavior of the *E*-filter to range from linear to highly nonlinear. We have found that the *E*-filter can be simply implemented and has several properties which make it attractive for use in edge detection problems. We have also shown the superiority of the *E*-filter to scale-space filtering for edge detection in one-dimensional systems.**

Index Terms—**Homomorphic filter, nonlinearity, scale-space filter.**

I. Introduction

Interpretation of measurements made by remote sensing devices must contend with the spatial and/or temporal resolution limitations of the measurement system and, of course, the presence of noise. This problem is shared by many disciplines. In the field of two-dimensional image processing, a common task in the chain of interpreting images is the identification of edges [3]. Once identified, the edge description of an image can be incorporated in various interpretation systems. In our applications, we have a similar task: identification of the edges between beds of different lithology. One difference is that often our data are one dimensional; they describe a physical measurement made as a function of depth in an oil well, for example. Again, once the bed boundaries have been identified, they serve as input to any number of schemes to improve the resolution of the measurements.

One difficulty in any edge-detection system is, of course, that sharp transitions which define the edges are sharp because of their high-frequency content, and any linear filtering done to suppress noise will also tend to blur the important transitions. Some filtering is necessary, however, due to the fact that edge detection relies essentially on derivatives of the data. A number of alternative schemes have been proposed to overcome this difficulty; median filtering [6] is one example. A drawback to median filtering in the context of edge detection, however, is that lines in two-dimensional images or spikes in one-dimensional curves do not survive the filtering process. In this correspondence, we shall describe some of our work with a generalization of the *E*-filter originally proposed by Moore and Parker [5], which we have found to possess several attractive properties for use in the edge-detection problem.

In the next section, then, we describe the *E*-filter and some of its relevant properties and propose a simple generalization. Section III discusses some applications of the generalized *E*-filter to edge detection in one-dimensional systems using an approach based on the generalized *E*-filter. We compare this to the method of scale-space filtering as proposed by Witkin [8]. The use of the *E*-filter in this application eliminates several difficulties encountered with the implementation of scale-space filtering.

II. Generalized *E*-Filter

In this section, we discuss the properties of the *E*-filter and propose a more general form than was considered originally by Moore and Parker [5]. The *E*-filter is in the class of nonlinear homomorphic filters studied by Oppenheim *et al.* [7]. The *E*-transform of an input signal $f(x)$ is defined as follows:

$$\hat{f}(e(x)) = f(x), \tag{2.1}$$

$$e(x) = \int_0^x \sqrt{1 + (\dot{f}(\xi))^2}\, d\xi, \tag{2.2}$$

where $\dot{f}(\xi) = df(\xi)/d\xi$. The input signal $f(x)$ is mapped into the *E*-domain with the amplitudes preserved.

The important property of this transformation is that large values of the gradient $\dot{f}(\xi)$ stretch the signal in the *E*-domain, thereby mapping "significant" structures to lower frequency. A low-pass filter applied to the transformed signal will therefore pass the significant structures unaffected. One difficulty which arises in applying *E*-filters to real problems is the design of the low-pass filter. For linear processing, one normally looks at the frequency content of typical input signals for the application, and chooses filter parameters, such as cutoff frequency, accordingly. The choice of cutoff frequency for a nonlinear process would appear to be problematic.

In order to obtain some control over the nonlinearity, we introduce a parameter p into the *E*-transform as follows:

$$e(x; p) = \int_0^x \sqrt{1 + p\left(\dot{f}(\xi)\right)^2}\, d\xi. \tag{2.3}$$

The definition of "significant" structure can now controlled by the parameter p. When the product $p\dot{f}(\xi)^2$ is small compared to 1, the *E*-transform reduces to the identity map and the *E*-filter to just the

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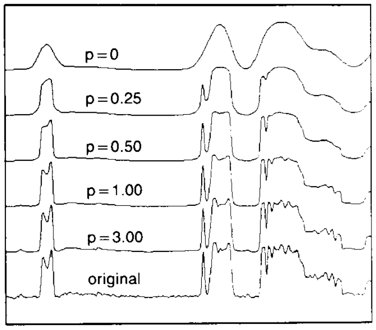

Fig. 1. Gamma-ray log. The original signal represents the measurement of natural formation radioactivity as a function of depth in the well. The parameter p controls the nonlinearity of the generalized E-filter; the nonlinearity increases as p increases.

low-pass filter. When the product is large with respect to 1, the nonlinear aspects become more pronounced; eventually, the low-pass filter becomes ineffective.

We demonstrate the effects of varying p in Fig. 1. The signal in this example is a measurement of the natural formation radioactivity as a function of depth in the borehole, known as a gamma-ray log. The noise in this measurement is dominated by counting statistics. The top trace in the figure depicts the filtered signal with the E-transform turned off. Moving toward the bottom of the figure, p is increasing and the Gaussian filter is kept constant. For large values of p, the filtered signal approaches the original input signal, displayed at the bottom of Fig. 1.

We should note here that the E-transform maps regularly sampled data into an irregularly sampled domain. One method of handling this problem is to interpolate to a regular interval, filter, and then interpolate back. Using the approximated Gaussian filters developed by Burt [2], we can efficiently obtain the set of signals convolved with various sizes of Gaussian filters. An alternative scheme, which we have developed, is to assume that the sampled data define a piecewise-linear function, i.e., in the lth interval, we write

$$f^{(l)}(e) = f(e_l) + (e - e_l)\frac{f(e_{l+1}) - f(e_l)}{e_{l+1} - e_l}. \tag{2.4}$$

We apply an analytically defined filter such as a Blackman–Harris window [4], which can be written as follows:

$$b(t) = b_0 + \sum_{n=1}^{3} b_n \cos\left(\frac{2n\pi t}{T}\right), \tag{2.5}$$

for $b_0 = 0.35869$, $b_1 = -0.48829$, $b_2 = 0.14128$, and $b_3 = -0.01168$. The Blackman–Harris window has a weight of $b_0 T$ over the interval $[0, T]$; so we write the filtered signal evaluated at the lth sample $R(e_l)$ as follows:

$$b_0 TR(e_l) = \int_{e_l - T/2}^{e_l + T/2} f(t)\, b(t - e_l + T/2)\, dt. \tag{2.6}$$

These integrals are readily computed and reduce to evaluating sums of cosine terms. This technique avoids the two interpolation steps required in the standard application of E-filters and improves the numerical efficiency. In timings of this scheme on a VAX 8650 processor, 500-point samples were filtered in roughly 1 CPU s. A simple median filter of the same length runs approximately an order of magnitude faster.

III. Edge Detection in One-Dimensional Systems

Witkin [8] proposed some time ago that convolution of a signal with Gaussian functions of various widths can serve to identify significant structures in the signal, without resort to *a priori* information. The basic idea is to convolve the signal with Gaussian functions of increasing width, mapping the function into the "scale space." At large values of the Gaussian width, only a few structures remain, and they are defined to be significant. They are identified by computing the zero crossings of the second derivatives of the scaled signal. The structure boundaries are then tracked to small values of the Gaussian width for accurate placement of the edges.

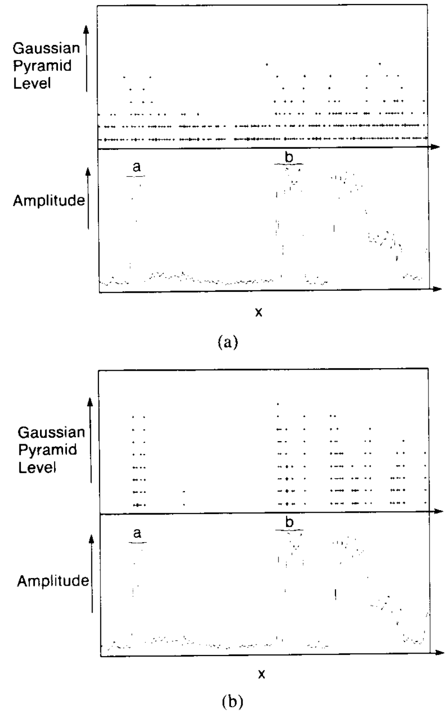

Fig. 2. Edge positions in the gamma-ray log. (a) Linear filtering. The points mark the positions of edges detected in the gamma-ray log from Fig. 1 as a function of Gaussian width. (b) E-filtered results. The nonlinearity parameter was set to be $p = 1$.

In practice, there are some difficulties in implementing the scale-space filter as described above. Computation of the second derivatives of sampled data is a noisy process. In most of our applications, we have looked at extreme points of the first derivatives rather than zero crossings of the second derivatives. Edges are defined by extreme values of the first derivative greater than some threshold value. A more severe problem is tracking the edge; as the Gaussian widths decrease, more and more edges are detected, and identification of the significant structure becomes problematic. Witkin [8], in fact, devotes a significant amount of time to developing a hierarchical tree to assist in following the edge. The problem is illustrated in Fig. 2(a) where we have plotted the edges detected in the gamma-ray log from Fig. 1. In this example, we have set $p = 0$ to turn off the E-filter. Note that as the Gaussian width increases, the edge positions migrate some distance from the original location, especially around the structure marked "a" and "b."

The edge migration can be readily explained. For the sake of simplicity, we use the second derivative/zero-crossing scheme instead of the first derivative/extremum scheme mentioned above for detecting edge positions. We define the unit step function $\theta(x - x_1)$ to be zero for $x < x_1$ and one for $x > x_1$. The second derivative of the convolution of this function with a Gaussian is decribed as follows:

$$G_{xx}(x, \sigma) * \theta(x - x_1) = \frac{x_1 - x}{\sigma^2} G(x_1 - x, \sigma), \tag{3.7}$$

where $G(x, \sigma) = 1/\sqrt{2\pi}\sigma \exp(-x^2/2\sigma^2)$ and $G_{xx}(x, \sigma)$ is the second derivative of $G(x, \sigma)$ with respect to x. The edge is located by requiring this expression to vanish, which occurs when $x = x_1$, independent of Gaussian width σ. In the case of the boxcar function consisting of two step functions $f(x) \equiv \theta(x - x_1) - \theta(x - x_2)$, the situation changes. The second derivative of the convolved signal becomes

$$G_{xx}(x, \sigma) * f(x) = \frac{x_1 - x}{\sigma^2} G(x_1 - x, \sigma) - \frac{x_2 - x}{\sigma^2} G(x_2 - x, \sigma). \tag{3.8}$$

In the limit where the edges are far apart compared to the Gaussian width ($|x_1 - x_2| \gg \sigma$), this vanishes approximately for $x = x_1$ and $x = x_2$. In the other case ($|x_1 - x_2| \ll \sigma$), this vanishes for $x = (x_1 + x_2)/2 \pm \sigma$. Consequently, scale-space filtering will suffer from edge migration due to the coherent interference of neighboring structures.

Using the generalized E-filter, we can avoid these coherent interferences to some extent. In Fig. 2(b), we display the results of a calculation of edge positions of the gamma-ray log with $p = 1$, that is, with the E-filter active. It is apparent that the positions of the edges do not depend on the scale width. This results from the ability of the E-filter to stretch the signal in the E-domain so that there is no interference from nearby structures. Using the boxcar function described above, we can examine this property in detail. In Fig. 3, the boxcar function and its generalized E-transformed signal are displayed. We can easily see that the points B and D go away from A and C, respectively, when either the boxcar's height h or the parameter p is increased. In this simple situation, we can calculate the range of p to obtain the correct edge positions as a function of σ given the width $w(= |x_1 - x_2|)$, h. The second derivative of the Gaussian convolved signal on the E-domain can be calculated using integration by parts:

$$\begin{aligned} F(e; p, \sigma) &\equiv G_{ee}(e, \sigma) * \hat{f}(e; p) \\ &= \frac{1}{p}\big(G(e - x_1, \sigma) - G(e - x_1 - ph, \sigma) \\ &\quad - G(e - x_2 - ph, \sigma) + G(e - x_2 - 2ph, \sigma)\big). \end{aligned} \tag{3.9}$$

Our concern is now the uniqueness of the zero crossings of this function and their range of existence on the E domain.

As for the uniqueness, Babaud *et al.* [1] and Yuille *et al.* [9] proved that extra zero crossings are never created as the scale increases using a Gaussian filter. In our case, when $\sigma = 0$, zero crossings can be defined at $e = x_1 + ph/2, x_2 + 3ph/2$, so the number of zero crossings is just two for any $\sigma > 0$. Moreover, these two zero crossings move toward $\pm\infty$, respectively, as σ increases.

Now that we have understood the behavior of the zero crossings, it is easy to derive the condition to obtain the correct edge positions. As we can see in Fig. 3, any point in the interval $[x_1, x_1 + ph]$ (respectively, $[x_2 + ph, x_2 + 2ph]$) on the E-domain is mapped back to a single point x_1 (respectively, x_2) on the original domain, which is the correct edge position. Therefore, the condition to be derived is that the zero crossings of (3.9) stay in these intervals on the E-domain, that is, simply, $F(x_1; p, \sigma) > 0$ and $F(x_1 + ph; p, \sigma) < 0$. Due to the symmetric nature of $F(e)$, these imply that $F(x_2 + ph; p, \sigma) < 0$ and $F(x_2 + 2ph; p, \sigma) > 0$. We cannot derive the range of p to satisfy these inequalities analytically, but numerically. Fig. 4 shows the lower bound of p to satisfy these with changing σ, w for fixed h. It is apparent that the range of p gets narrower with decreasing w and increasing σ.

To confirm the effect of the parameter p in another context, we applied the generalized E-filter to the Gaussian signal itself with changing p and the smoothing width, and we then checked zero-crossing positions. This is illustrated in Fig. 5. As we can see, increasing the parameter p has the effect of "remigrating" the zero-crossing positions toward the correct edge positions, as does the previous boxcar example.

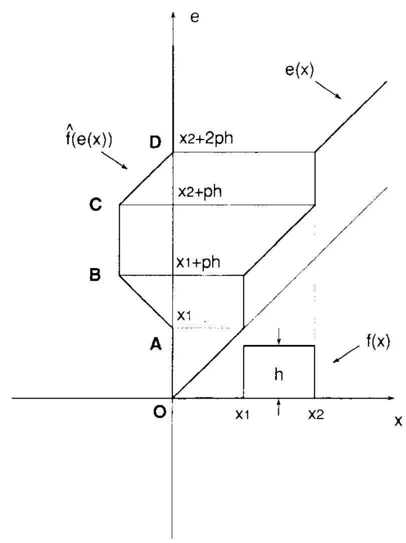

Fig. 3. Generalized E-transform of the boxcar function. The distance $\overline{AB}$ and $\overline{CD}$ is controllable by p. Any point in the interval $[x_1, x_1 + ph]$ and $[x_2 + ph, x_2 + 2ph]$ on the E-domain is mapped back to the points x_1 and x_2 on the original domain, respectively.

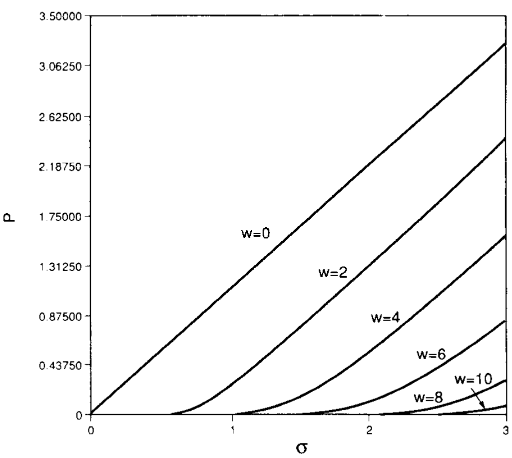

Fig. 4. The lower bound of parameter p to prevent the edge migration in the boxcar example with changing σ and w. h is fixed to 1 in this case. The acceptable range of p gets narrower as σ increases and w decreases.

With the use of the E-filter, it is therefore possible to simply filter with a large width and obtain the placement of the edge positions directly, without having to track the structures to small Gaussian widths. This is a significant savings in terms of computational effort. Although the generalized E-filter is somewhat more computationally intensive than simple linear filters, the scale-space algorithm no longer needs to be iterated. Moreover, there is no need to track the edges.

Use of the generalized E-filter also solves another difficulty found with scale-space filtering, which is called the "masking" effect, and is related to the loss of high-frequency information for large values of the Gaussian width. In Fig. 2, we see an illustration of the problem. The two structures marked "b" are separated by a

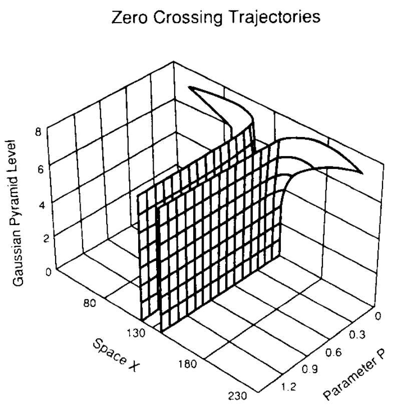

Fig. 5. Edge migration. The edge positions of a Gaussian shape signal are shown as a function of both Gaussian width and nonlinearity parameter p. With the E-filter active ($p > 0$), the edge positions remain stable.

relatively small gap. Standard linear filtering, as used in the scale-space filter, cannot resolve them, and only one structure is identified. When the edge detection is based on the E-filter approach, the two structures remain separated, even for large Gaussian widths, and both are correctly identified.

IV. Conclusion

We have described a simple generalization of the E-filter proposed by Moore and Parker [5], and have shown its utility in one-dimensional edge detection problems. We have found that the E-filter method is superior to scale-space filtering in these systems in the sense of the edge position migration. In particular, significant but narrow structures are not adversely affected by the E-filter; they are readily identified. We have not addressed the issue of very noisy data. In that case, it will be difficult to provide any improvement over what can be obtained with matched filters. In the cases we have shown, however, the E-filter provides an attractive alternative to more standard linear techniques.

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Symbolic Construction of a 2-D Scale-Space Image

ERIC SAUND

***Abstract*—This correspondence offers a *symbolic* approach to constructing a multiscale primitive shape description for 2-D binary (silhouette) shape images. In contrast to contour or region smoothing techniques, *grouping* operations are performed over collections of tokens residing on a scale-space blackboard. Two types of grouping operations are identified that, respectively, 1) aggregate edge primitives at one scale into edge primitives at a coarser scale, and 2) group edge primitives into partial-region assertions, including curved-contours, primitive-corners, and bars. Procedures to perform these computations are presented.**

***Index Terms*—Image understanding, machine vision, multiscale, perceptual grouping, primal sketch, scale-space, shape representation, symbolic token grouping.**

I. Introduction

The shapes of naturally occurring objects characteristically involve spatial events occurring at a multitude of spatial scales. Shape details appearing at smaller scales are situated in relation to one another by the spatial structure emergent at larger scales. It is important to make explicit the multiscale structure of a shape object[1] because important distinguishing characteristics or features may occur at any scale. For this reason, one widely cited goal for early visual shape processing is to construct a description of a shape across a variety of scales [2], [6], [9], [10], [14], [22], [24], [28], [35]. From these descriptions may be extracted important primitive shape events to be used by later stages devoted to object recognition or other visual tasks. This correspondence is concerned with building multiscale shape descriptions of two-dimensional binary (silhouette) shape images in terms of edge and region shape primitives.

Currently available techniques for multiscale shape analysis are of two basic types: contour-based smoothing and region-based smoothing. Both of these approaches are based on the application of a numerical smoothing operator uniformly to some one-dimensional (contour-based) or two-dimensional (region-based) array of shape data. The operator is typically characterized by a size or width parameter indicating the degree of smoothing performed, and hence the scale of the result. As will be illustrated, at coarse scales both contour-based smoothing and isotropic region-based smoothing approaches fail to capture in a consistent manner important aspects of the geometric structure inherent to shape objects. The prospects for oriented region-based filters are uncertain.

This correspondence describes a fundamentally different approach to extracting a primitive shape description at multiple scales. The approach is based on grouping of shape tokens in the style of the Primal Sketch [19]. A shape token is a packet of information making explicit the pose (location, orientation, and scale) plus other data about a fragment of shape such as a local figure/ground boundary. Token grouping may be considered a *symbolic* computation

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[1]We refer to a figure whose shape we are analyzing as a *shape object*.