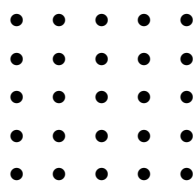# 135A Practice Final Problems
Now or Never

1. Social security numbers in East Datoka are comprised of 7 binary digits followed by two letters. How many cars can this system accommodate? What if governor Weissei decides to use decimals for the digits but does not allow any letters or digits to repeat?

2. Draw a 5 times 5 grid of dots

$$\begin{matrix} \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \end{matrix}$$

   Starting from the bottom left and ending at the top right draw a path from the bottom left to the top right dot following the rule that you can only take one step up or one step right at a time. How many such paths are there? Now put a number of pieces of candy on the dots as shown in this diagram

$$\begin{matrix} 4 & 3 & 2 & 1 & 0 \\ 3 & 2 & 1 & 0 & 1 \\ 2 & 1 & 0 & 1 & 2 \\ 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 3 & 4 \end{matrix}$$

   Now pick a path at random and eat all the candy you find en route. How many pieces of candy do you expect to eat?

3. Julian Schwinger invests in the stock market. His return in any given year is measured by a Normal$(100000, 50000)$ random variable S. A detailed fluctuation analysis of his portfolio requires that he compute the expectation of the new random variable $T = S^8$. Use Schwinger's trick to compute this expectation for him.

4. There are m monkeys and g gorillas in a jungle. Diane Fossey captures r apes (an ape is defined as a monkey or a gorilla). What is the chance that

she captures no monkeys? One monkey? Two monkeys? k monkeys? Only monkeys? Prove the following formula

$$\binom{a+b}{c} = \sum_{k=0}^{c} \binom{a}{k}\binom{b}{k-c}$$

5. In the previous problem, now suppose (despite her completely random ape trapping technique), the first s apes captured by Diane are monkeys. Now compute the chance that she captures k monkeys.

6. Explain why choosing randomly a number between 0 and 1, expressed in base 6 is like rolling a dice infinitely many times. Suppose your close friend Manny Times rolls a dice infinitely often. On the first roll you give him $k/6$ of your savings account where $k + 1$ is the value shown on the dice. On the second roll you given him $k/6^2$ of the original balance of your savings account, and so on. How likely is Manny to end up with over half your savings?

7. Yahtzee is a popular game played by rolling 5 dice. A full house occurs when there is one pair and three dice alike (ex. 33322, 44466). You roll all five dice twice. Let H be the random variable measuring the number of full houses rolled. Let T be the number of times your roll two pair (ex. 11223 or 22446 etc....). Tabulate the joint probability mass function of (H, T) as well as the marginal mass functions for each variable. Indicate what checks you could make on the accuracy of your computations.

8. The length of Saharan rattlesnakes is a random variable L (measured in meters) with distribution

$$p(l) = \begin{cases} 2a - al & 0 \le l \le 2 \\ 0 & \text{else} \end{cases}$$

Find a. You find a rattlesnake in the Sahara. How long do you expect it to be? Your friend returns home from a trip to the Sahara with a rattle snake hidden in their sleeping bag and says "its at least a meter long". How long do you expect this snake to be?

9. Discuss how any approximations you know for the Binomial(n, p) distribution that you happen to know focusing on their range of validity. Extra credit if you can derive these.

10. Design a random experiment to measure π. Make an estimate of the accuracy of your design.

11. Randomly fold a square sheet of paper. How likely is it that the fold goes through two adjacent edges? Compute the expected area of the resulting figure if it does. Repeat the above for the case that the fold goes through opposite edges. Now compute the expected area for any random fold. Finally, what is the expected area if you start the fold through a corner?